Thermal photons and dileptons: viscous corrections

NC STATE UNIVERSITY

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2011 Workshop: Thermal Photons and Dileptons

Brookhaven National Laboratory

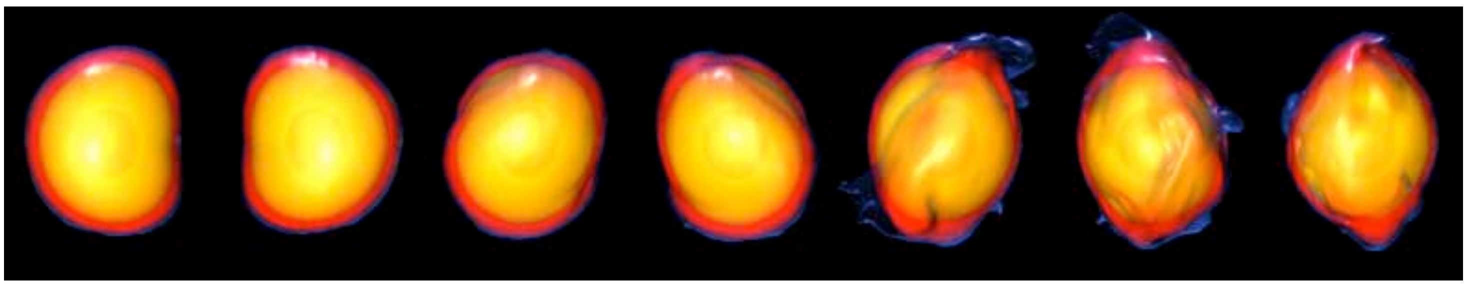
Outline

1. Introduction to bulk viscosity
2. Shear & Bulk viscosity in heavy - ion collisions
3. Perspectives on electromagnetic probes

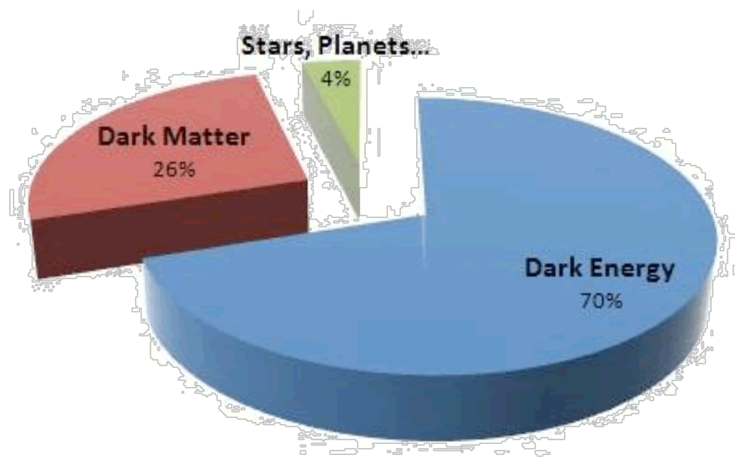
Introduction to bulk viscosity

Why study bulk viscosity in HIC?

- Damping of r-mode instability in rotating neutron stars:



- Bulk viscous cosmology:



Introduction to bulk viscosity

1. Consider uniform rarefaction

2. The density and energy density of the system will drop
3. The pressure also drops, but more than expected from

$$p = p_o(\epsilon, n_i)$$

   and the bulk viscosity characterizes this additional pressure shift.

4. The opposite scenario holds for a system in uniform compression.

Acoustic Spectroscopy

1. Sound attenuation can be related to the viscosity via generalized Stoke's law

$$\alpha = \frac{2}{3}\frac{\eta + \zeta}{nc_s^3}\omega^2 \qquad \omega\tau_\nu \ll 1$$

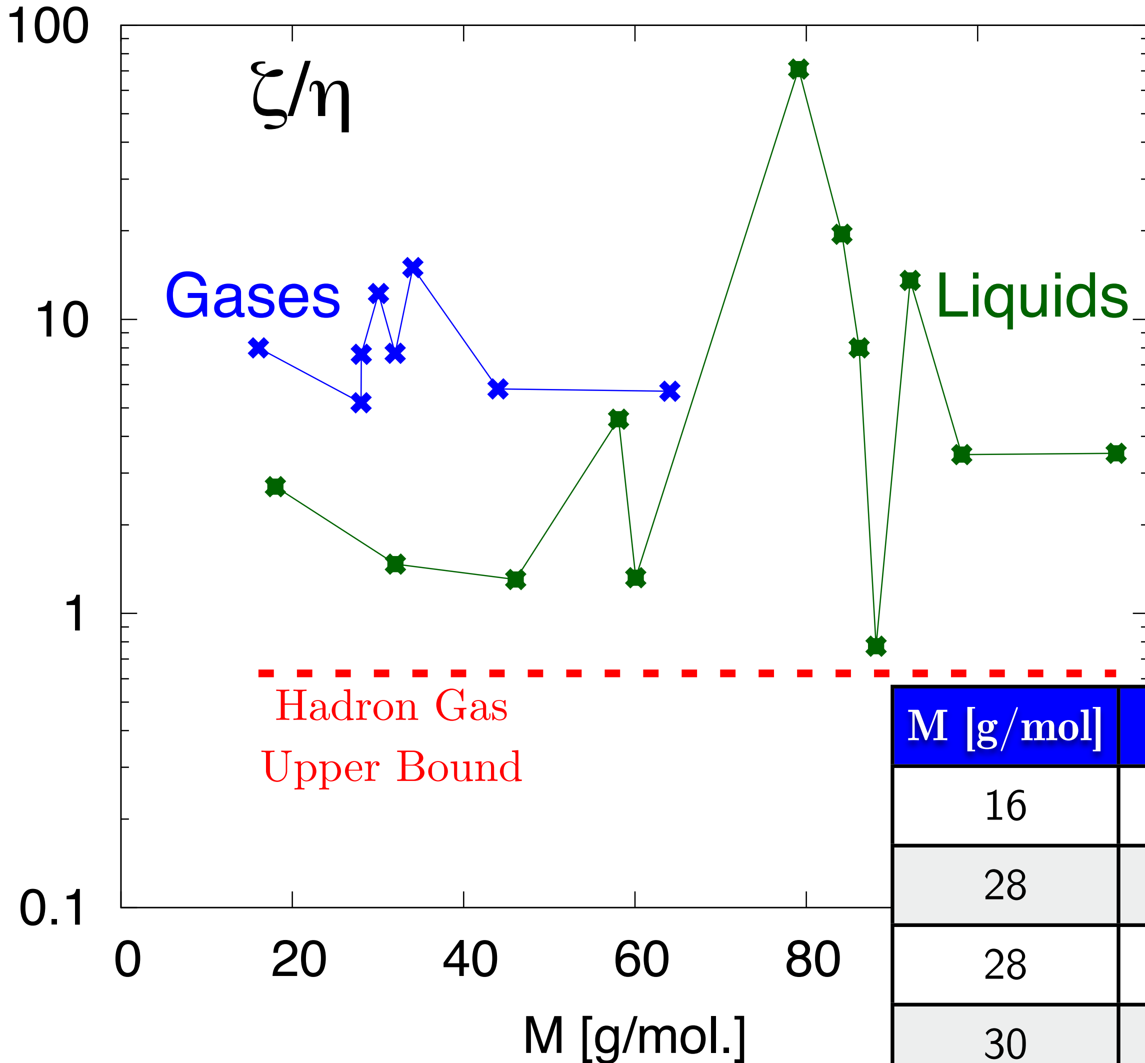
Origins of Bulk viscosity

1. Non-zero mean free path
2. Non-equilibrium chemistry
3. Internal DOF: vibrational, rotational
4. Dynamic mean fields

Phenomenon	# of Collisions
Translation	~ 10
Rotation	~ 10
Vibration	$\sim 10^4$
Dissociation	$> 10^4$

Viscosity is determined by the slowest process to equilibrate

Bulk and shear are independent quantities sensitive to different physics...



M [g/mol]	Gas
16	CH_4
28	N_2
28	C_2H_4
30	C_2H_6
32	O_2
34	H_2S
44	CO_2
64	SO_2

M [g/mol]	Liquid
18	Water
32	Methanol
46	Ethanol
58	Acetone
60	2-propanol
80	Pyridine
84	Cyclohexane
86	Hexane
88	1-pentanol
92	Toluene
98	Cyclohexanone
116	Butylacetate

Bulk Viscosity in heavy-ion collisions

KD, Thomas Schaefer, arXiv:1109:5181

Bulk Viscosity in heavy-ion collisions

1. Relaxation Time Approximation:

$$\zeta \approx 15\eta \left(\frac{1}{3} - c_s^2\right)^2$$

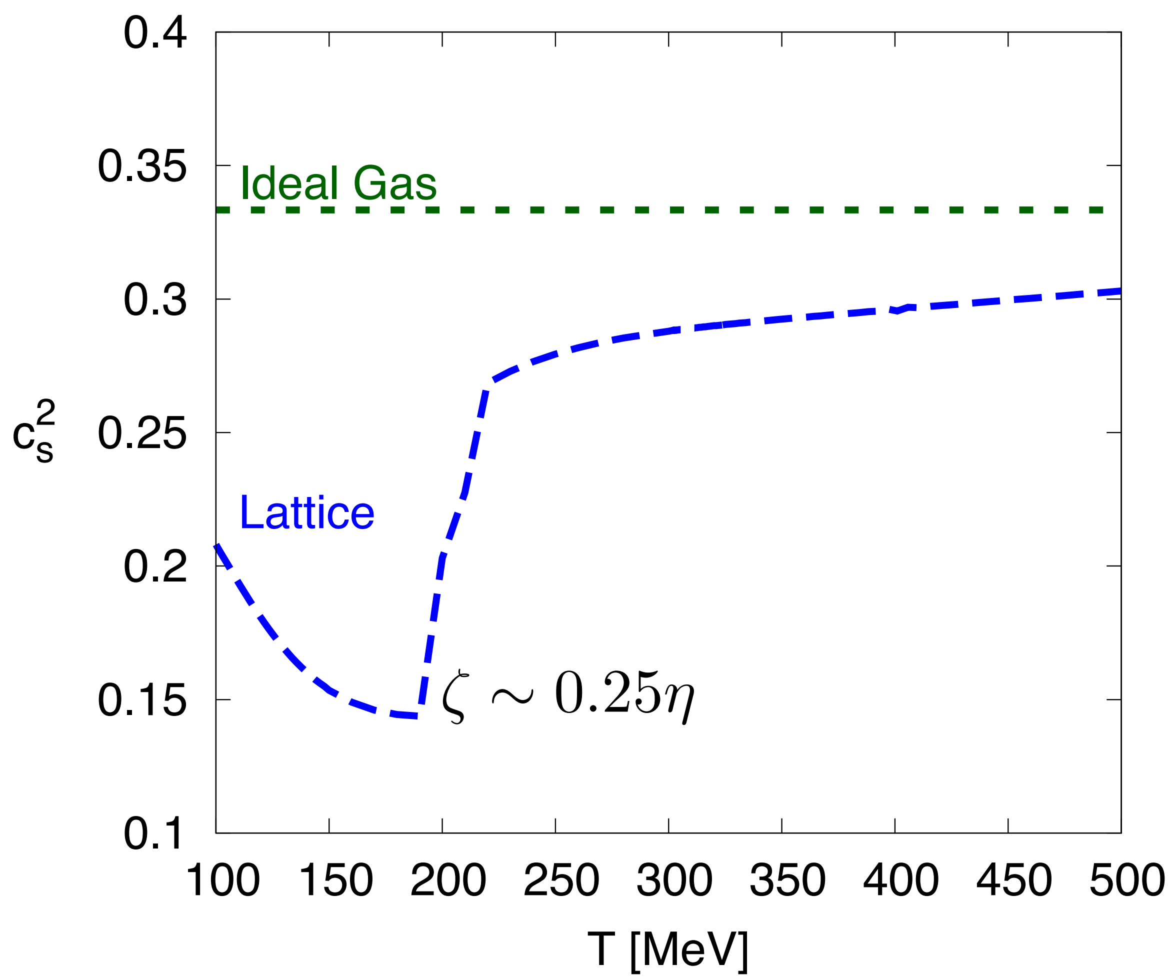
S. Weinberg, Astrophys. J. 168:175, 1971.

2. AdS/CFT:

$$\zeta \gtrsim 2\eta \left(\frac{1}{3} - c_s^2\right)$$

A. Buchel. Phys. Lett. B663:286-289, 2008.

Equation of state



Hydrodynamics

1. Equation of Motion:

$$\partial_\mu T^{\mu\nu} = 0$$

2. Stress-Energy Tensor:

$$T^{\mu\nu} = (\epsilon + \mathcal{P})\, u^\mu u^\nu + \mathcal{P} g^{\mu\nu} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}$$

3. In the LRF:

$$\begin{aligned}\pi^{ij} &= -\eta\left(\partial^i u^j + \partial^j u^i - \frac{2}{3}\delta^{ij}\partial_k u^k\right) = -\eta\sigma^{ij} \equiv -2\eta\langle\partial^i u^j\rangle\;, \\ \Pi &= -\zeta\partial_k u^k\end{aligned}$$

Freeze-out

1. The standard Cooper-Frye formula:

$$E_{\mathbf{p}} \frac{dN}{d^3p} = \frac{1}{(2\pi)^3} \int_\sigma f(E_{\mathbf{p}}) p^\mu d\sigma_\mu$$

2. must include viscous corrections of form

$$f(E_{\mathbf{p}}) = f_0(E_{\mathbf{p}}) + \delta f(E_{\mathbf{p}})$$

3. with the constraint

$$\delta T^{\mu\nu} = \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}} p^\mu p^\nu \delta f(E_{\mathbf{p}})$$

How does shear viscosity appear in spectra?

1. Viscous correction to equation of motion

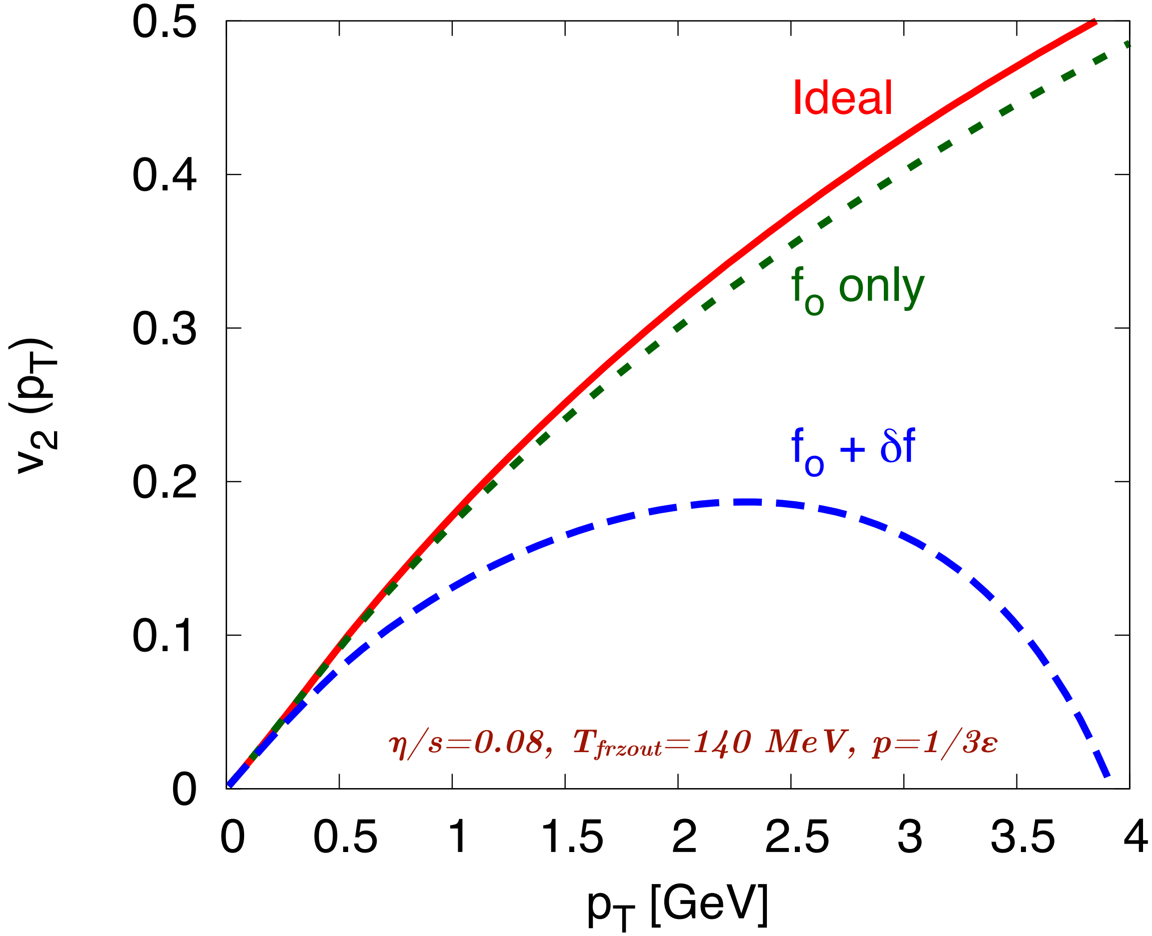
$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{where} \qquad T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu} - \eta \langle \partial^\mu u^\nu \rangle$$

2. Viscous correction to spectra

$$E\frac{d^3N}{d^3p} = \frac{\nu}{(2\pi)^3}\int_\sigma f_o + \delta f \; p^\mu d\sigma_\mu$$

$$\delta f = -\frac{\eta}{sT^3} \times f_0(p) p^i p^j \langle \partial_i u_j \rangle$$

How does shear viscosity appear in spectra?



Clearly we need to have a quantitative understanding of δf

Boltzmann Equation

1. Starting point will always be the Boltzmann equation

$$(\partial_t + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}})\, f(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}[f, \mathbf{p}]$$

$$v_{\mathbf{p}} = \frac{\mathbf{p}}{E_{\mathbf{p}}}\ , \qquad \mathbf{F} = -\frac{m}{E_{\mathbf{p}}}\partial_{\mathbf{x}} m = -\frac{\partial E_{\mathbf{p}}}{\partial \beta}\partial_{\mathbf{x}}\beta$$

2. along with this modified form of the stress tensor

$$T^{\mu\nu} = \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}} \left(P^\mu P^\nu - u^\mu u^\nu T^2 \frac{\partial m^2}{\partial T^2} \right) f(t, \mathbf{x}, \mathbf{p})$$

Relaxation Time Approximation: Shear

1. Take the simplest collision operator:

$$\mathcal{C}[f, \mathbf{p}] = \frac{f(\mathbf{p}) - n_{\mathbf{p}}}{\tau_R(E_{\mathbf{p}})} \qquad n_{\mathbf{p}} = \frac{1}{e^{\beta E_{\mathbf{p}}} \mp 1}$$

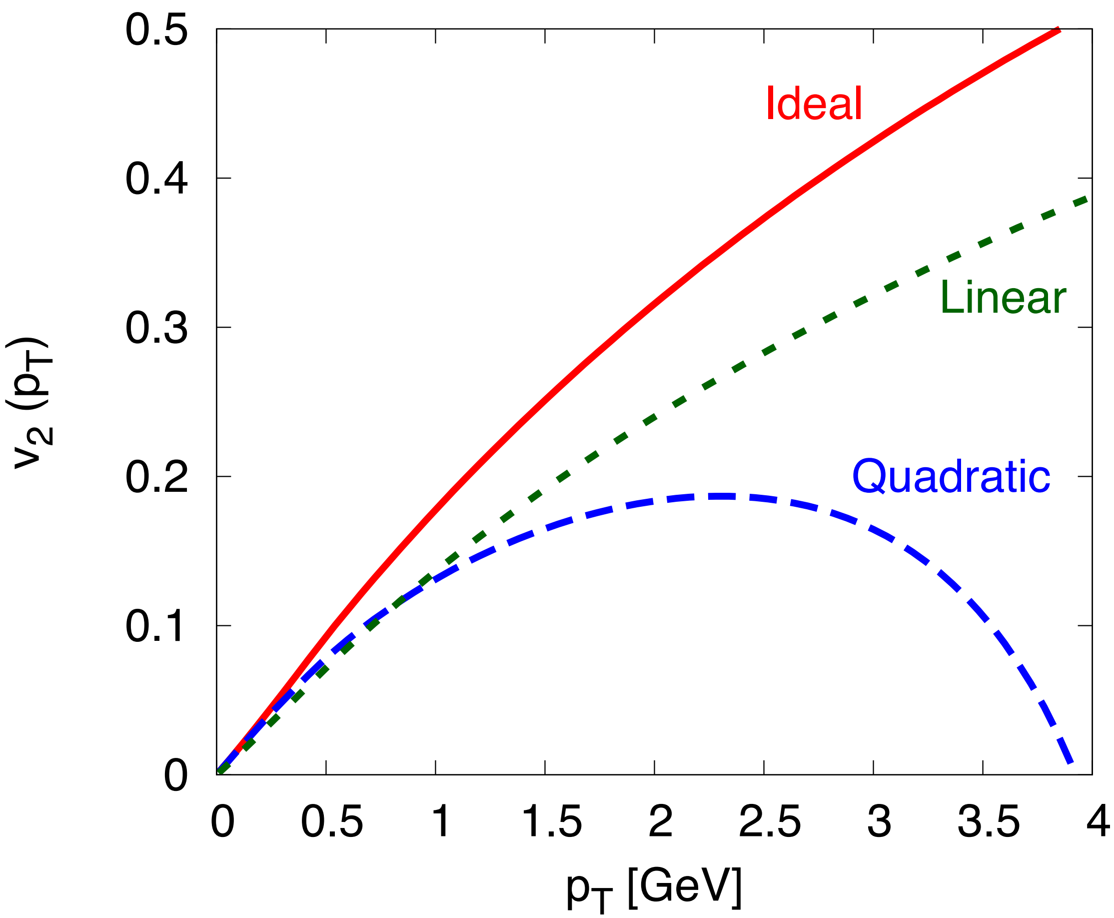
2. The viscous correction goes as

$$\delta f = -\frac{\tau_R(E_{\mathbf{p}})}{E_{\mathbf{p}} T} n_{\mathbf{p}} (1 \pm n_{\mathbf{p}}) \frac{1}{2} p^i p^j \sigma_{ij}$$

3. with the constraint

$$\delta T^{xy} = -2\eta \langle \partial^x u^y \rangle = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^x p^y}{E_{\mathbf{p}}} \delta f$$

How does shear viscosity appear in spectra?



The true form of δf is somewhere between linear and quadratic.

Weakly coupled pure-glue QCD

1. We start with the Boltzmann equation

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{2\leftrightarrow 2}[f] - \mathcal{C}^{1\leftrightarrow 2}[f]$$

2. and linearize it by substituting $f(p) = f_o(p) + \delta f(p)$

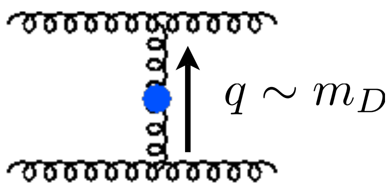
$$f_o \frac{p^i p^j}{T E_p} \langle \partial_i u_j \rangle = -\mathcal{C}^{2\leftrightarrow 2}[\delta f] - \mathcal{C}^{1\leftrightarrow 2}[\delta f]$$

3. The above integral equation can be inverted in order to obtain δf

Weakly coupled pure-glue QCD

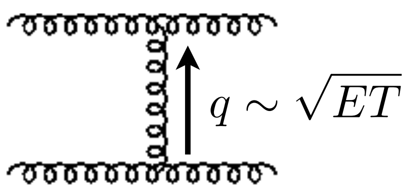
Three different modes of energy loss:

1. Soft Scattering



$$\frac{dp}{dt} \propto g^4 \log\left(\frac{T}{m_D}\right)$$

2. Collisional

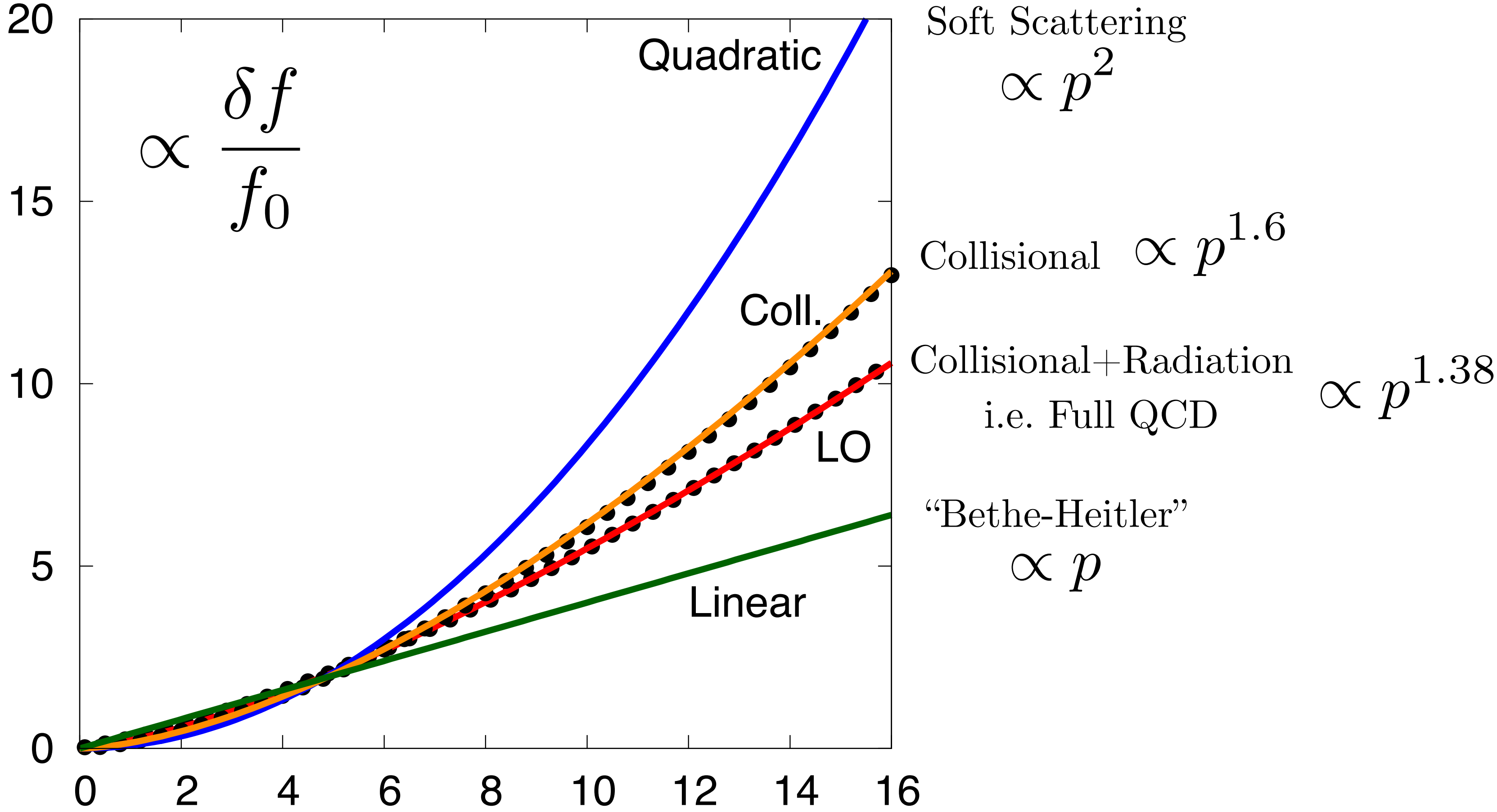


$$\frac{dp}{dt} \propto g^4 \log\left(\frac{p}{m_D}\right)$$

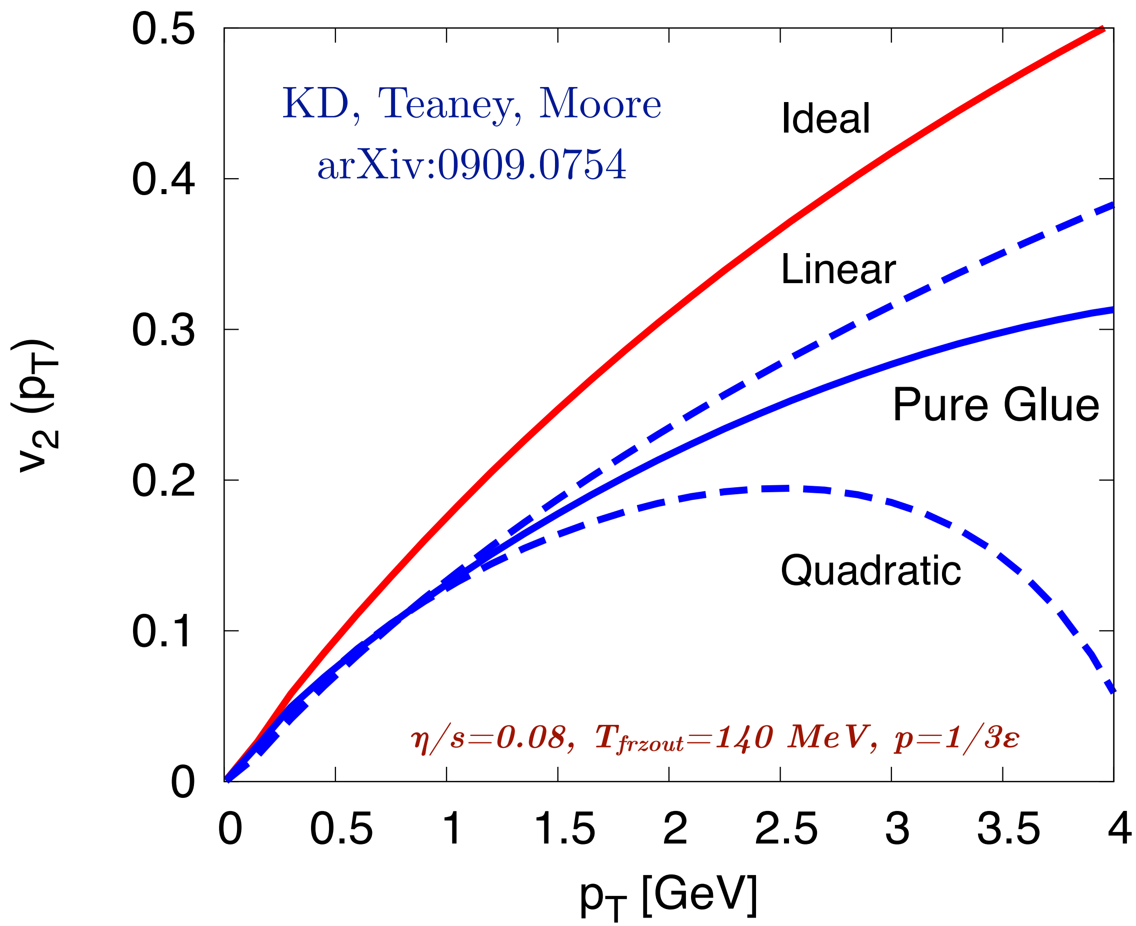
3. Radiative

$$\frac{\Delta p}{\Delta t} \propto g^2 \sqrt{\hat{q} E_p}$$

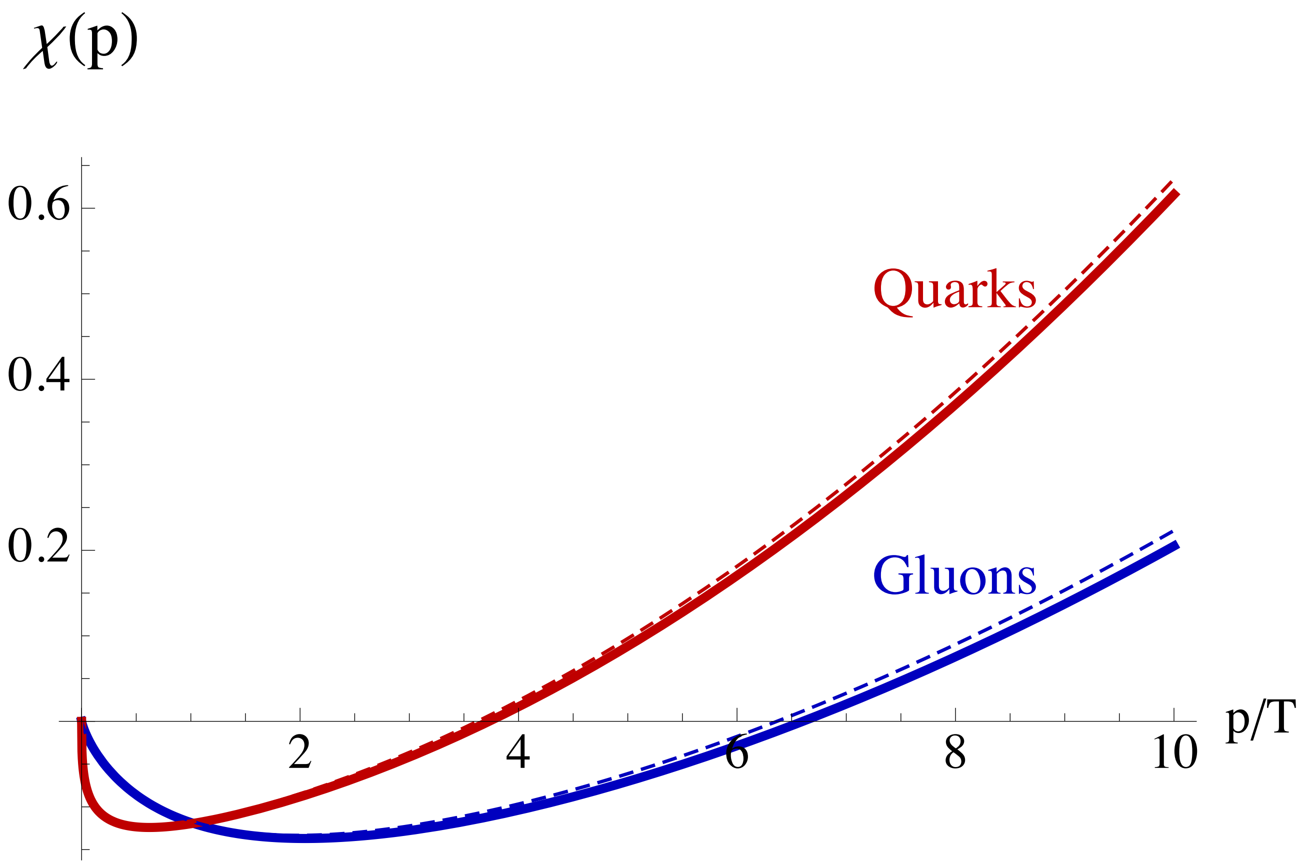
Shear Channel



Quark / gluon elliptic flow (w/ shear viscosity)



Bulk channel at leading log

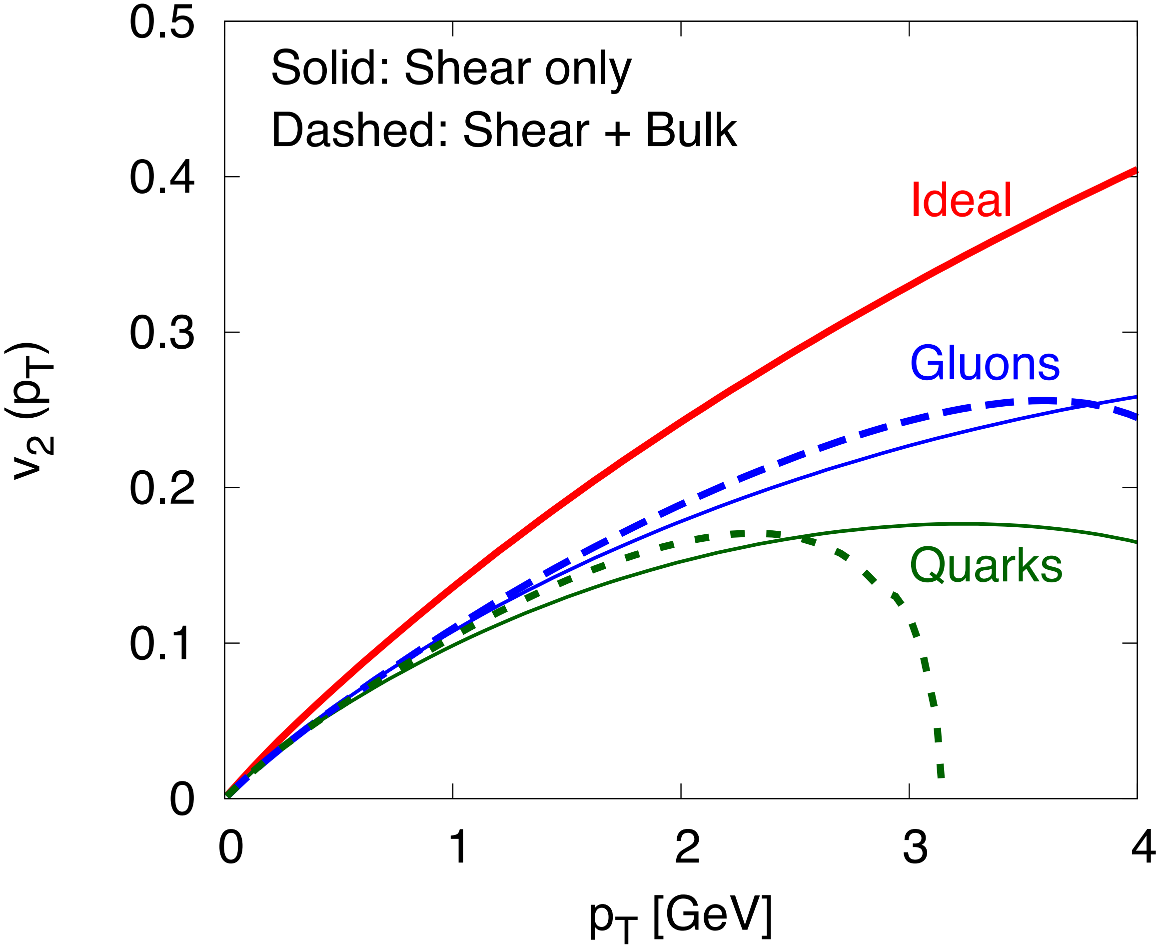


Same result as first done by:

Arnold, Dogan Moore, PRD 74:085021, 2006.

See also: Hong, Teaney PRC 82:044908, 2010.

Quark / gluon elliptic flow
(now including bulk viscosity)



Scalar / Pion gas

1. For scalar φ^4 and a pion gas we need to include inelastic processes

$$\left(\partial_t + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}}\right) f(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{2\leftrightarrow 2}[f, \mathbf{p}] - \mathcal{C}_{2\leftrightarrow 4}[f, \mathbf{p}]$$

2. and after linearizing around equilibrium solution we have

$$\frac{\beta}{E_{\mathbf{p}}}\left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}} \frac{\partial\left(\beta E_{\mathbf{p}}\right)}{\partial \beta}\right) = -\mathcal{C}_{2\leftrightarrow 2}[\delta f, \mathbf{p}] - \mathcal{C}_{2\leftrightarrow 4}[\delta f, \mathbf{p}]$$

See: Jeon, Yaffe, PRD 53:5799, 1996.
and Lu, Moore, PRC 83:044901, 2011.

Transition Rates

$$C_{2\leftrightarrow 2}[\delta f, \mathbf{p}] = \frac{1}{2!}\int_{\mathbf{k},\mathbf{p}',\mathbf{k}'} \Gamma_{\mathbf{pk}\to\mathbf{p}'\mathbf{k}'} \; n_{\mathbf{p}} n_{\mathbf{k}} (1+n_{\mathbf{p}'})(1+n_{\mathbf{k}'}) \times \left[\chi(p)+\chi(k)-\chi(p')-\chi(k')\right] \; ,$$

$$C_{2\to 4}[\delta f, \mathbf{p}] = \frac{1}{3!2!}\int_{\mathbf{k},\mathbf{p}',\mathbf{k}',\mathbf{q},\mathbf{q}'} \Gamma_{\mathbf{p}'\mathbf{k}\to\mathbf{pk}'\mathbf{qq}'} \; n_{\mathbf{p}} n_{\mathbf{k}'} n_{\mathbf{q}} n_{\mathbf{q}'} (1+n_{\mathbf{p}'})(1+n_{\mathbf{k}}) \times \left[\chi(p)+\chi(k)-\chi(p')-\chi(k')-\chi(q)-\chi(q')\right]$$

1. The $2 \leftrightarrow 2$ collision operator has an exact zero mode

$$\chi(p) = \chi_0 - \chi_1 E_p$$

Zero-mode solution

1. The two unknowns in the zero-mode solution

$$\chi(p) = \chi_0 - \chi_1 E_p$$

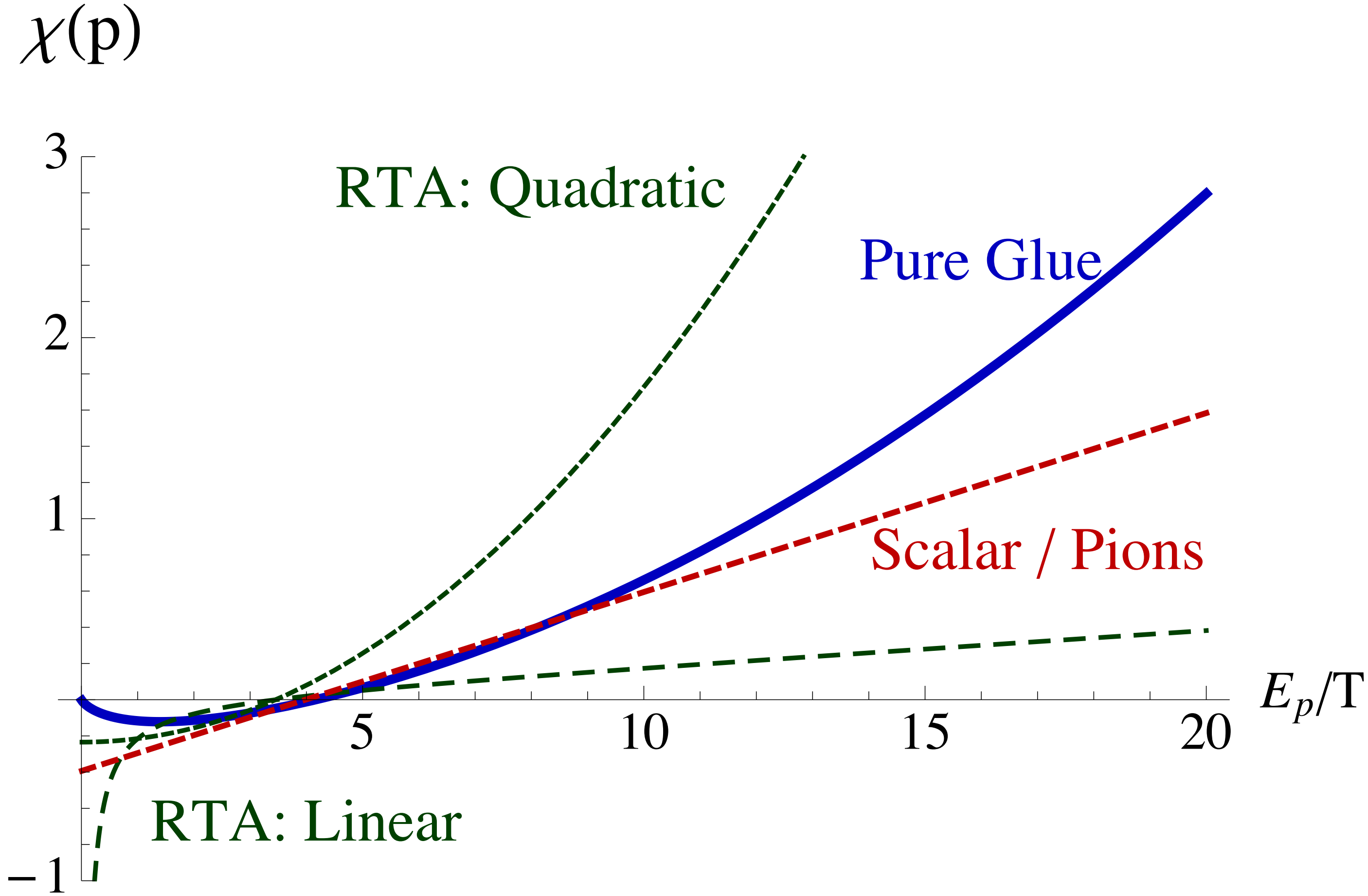
2. can be related to the bulk viscosity coefficient

$$\chi_0 = \frac{\zeta}{\mathcal{F}} \qquad \mathcal{F} \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}} \left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}} \frac{\partial \left(\beta E_{\mathbf{p}}\right)}{\partial \beta} \right) n_{\mathbf{p}} (1 + n_{\mathbf{p}})$$

3. and determined by landau matching

$$\delta\epsilon = 0 = \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}} \tilde{E}_{\mathbf{p}}^2 \delta f$$

Viscous corrections in bulk channel



Hadron Resonance Gas

1. Let us assume that in a HRG number change is the slowest process and the deviation from equilibrium is near the zero-mode,

$$\delta f^a(\mathbf{p}) = -n^a_{\mathbf{p}}(1 \pm n^a_{\mathbf{p}})\partial_k u^k \left(\chi^a_0 - \chi_1 E_{\mathbf{p}_a}\right)$$

   with $\mathrm{N}_{\mathrm{species}}+1$ unknowns.

2. One unknown is constrained by bulk viscosity

$$\zeta = \sum_a \nu_a \chi^a_0 \mathcal{F}^a \qquad \mathcal{F}^a = \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}_a}} \left(\frac{p^2}{3} - c_s^2 E^2_{\mathbf{p}_a}\right) n^a_{\mathbf{p}}(1 \pm n^a_{\mathbf{p}})$$

   and another by landau matching:

$$\delta\epsilon = 0 = \sum_a \nu_a \int \frac{d^3\mathbf{p}}{(2\pi)^3} E_{\mathbf{p}_a} \delta f^a(\mathbf{p})$$

The model

1. Many hadronic species are in relative chemical equilibrium

   For example: $\rho \longleftrightarrow 2\pi \,, \quad p\bar{p} \longleftrightarrow 5\pi$

   (See works of Pratt/Haglin, J.L. Goity, Song/Koch)

2. Let us take the simplest model

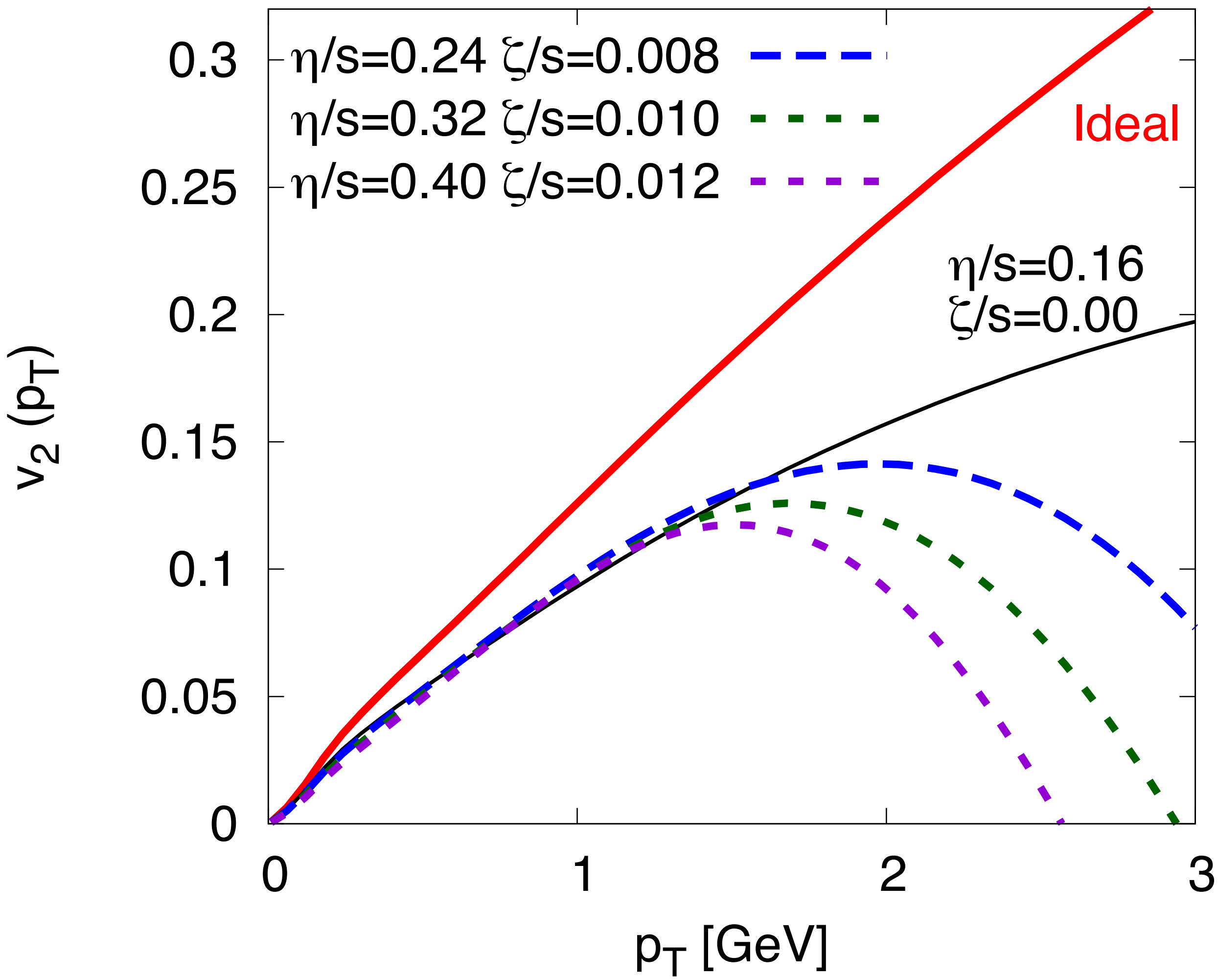
$$\chi_0^a = \begin{cases} \chi_0^\pi & \text{Pions} \\ C_m \times \chi_0^\pi & \text{Mesons} \\ C_b \times \chi_0^\pi & \text{Baryons} \end{cases} \qquad \begin{matrix} C_m \approx 2 \\ C_b \approx 2.5 \end{matrix}$$

3. and the bulk viscosity becomes

$$\zeta = \chi_0^\pi \sum_a \nu_a C_a \mathcal{F}^a \quad \text{where} \quad C_a = \begin{cases} 1 & \text{Pions} \\ C_m & \text{Mesons} \\ C_b & \text{Baryons} \end{cases}$$

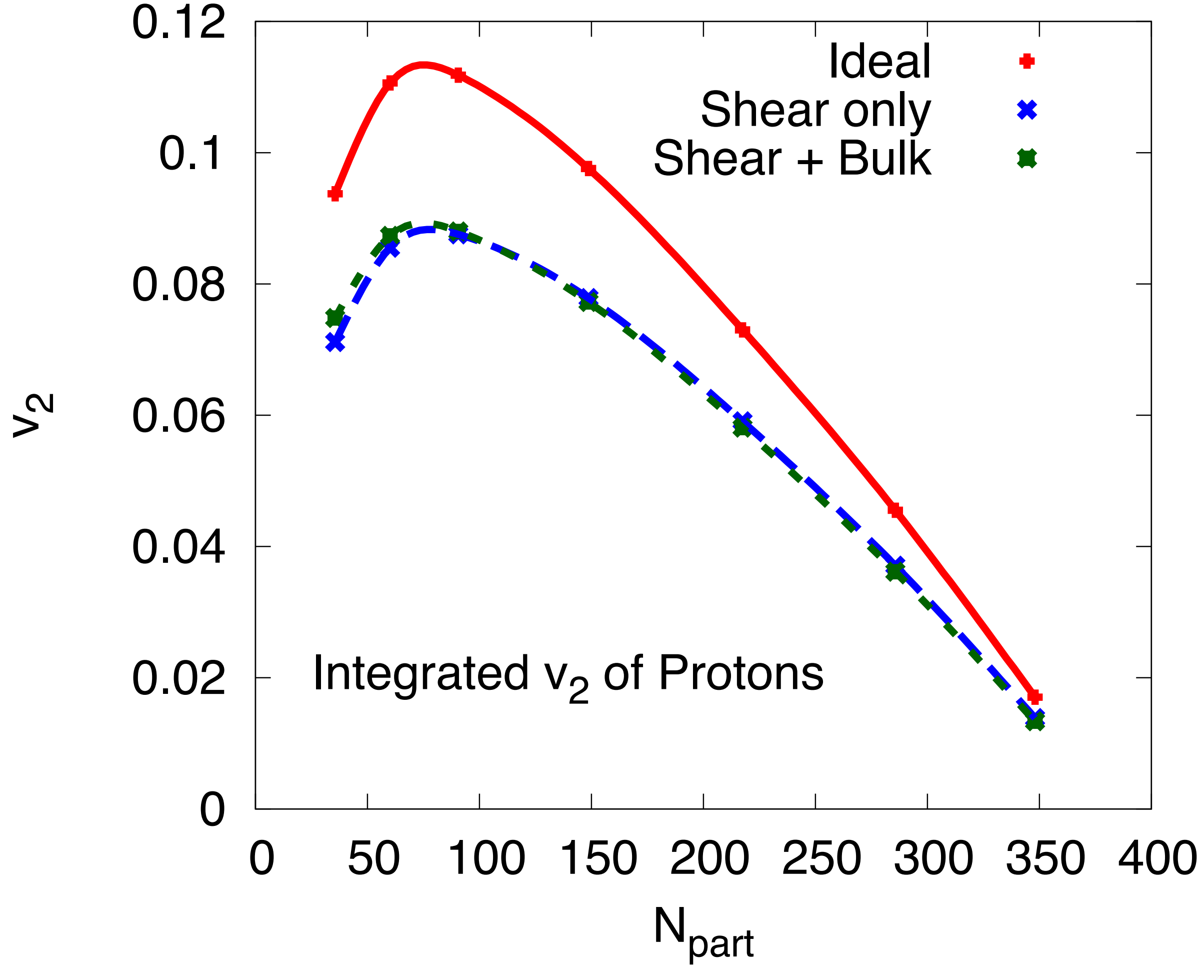
Phenomenological consequences

Differential v_2



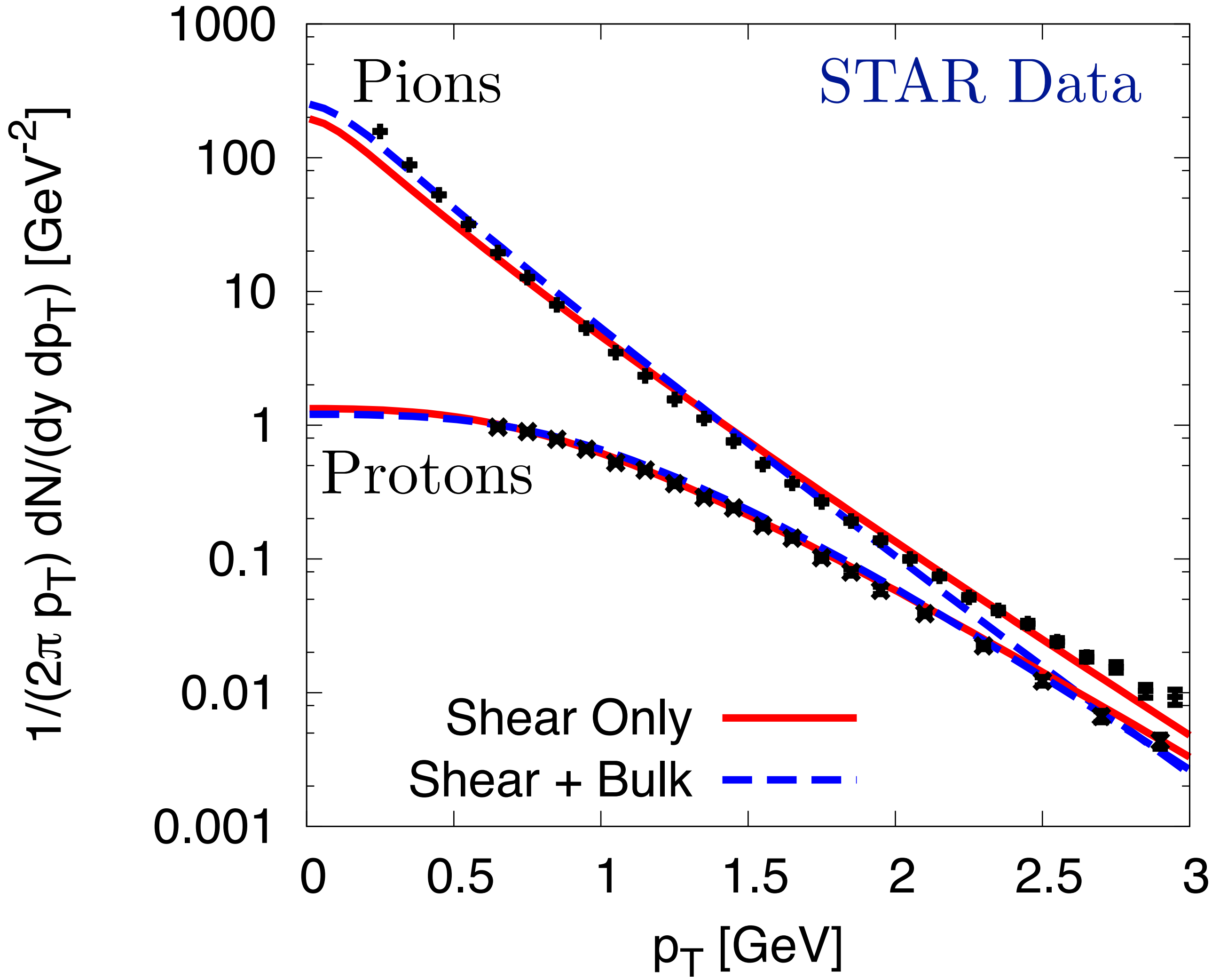
Ambiguity in separating bulk and shear in $v_2(p_T)$.

Integrated v_2

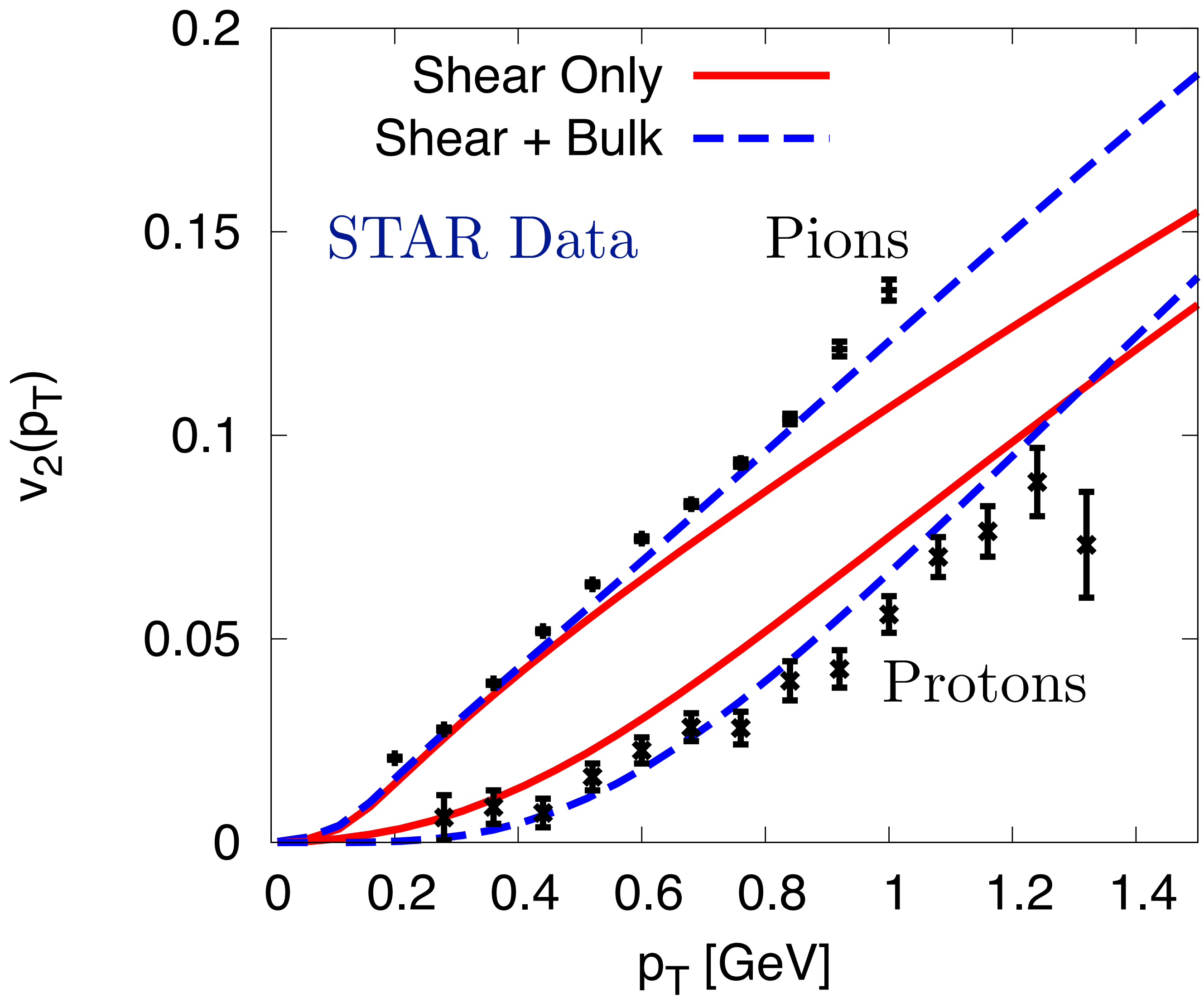


Integrated v_2 insensitive to bulk viscosity.

Effect of bulk viscosity on spectra



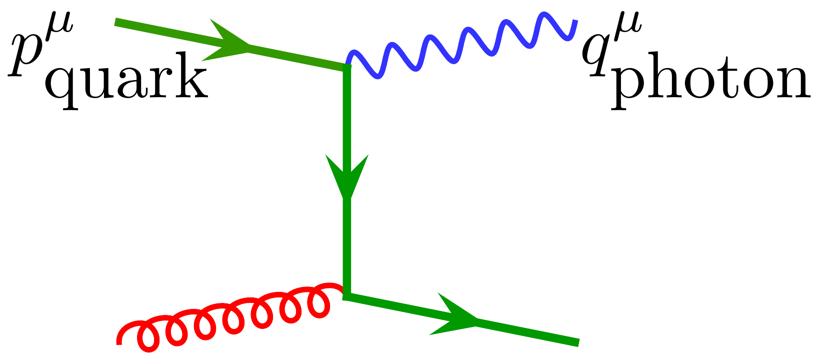
Effect of bulk viscosity on flow



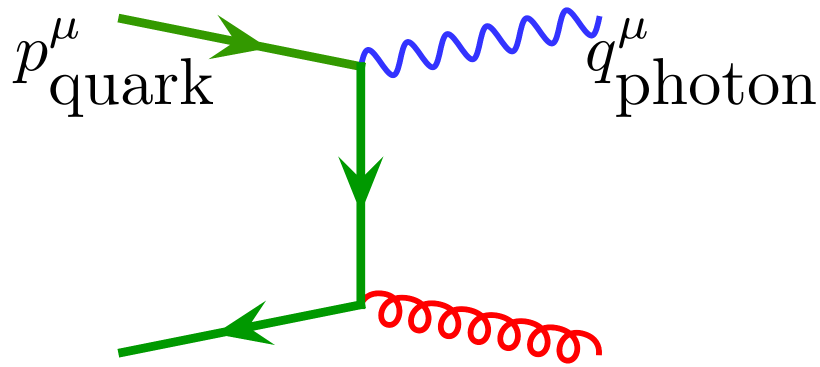
Perspectives on Electromagnetic Probes

Photon production at leading log

Compton:



Annihilation:



1. Photons are completely out of equilibrium.
   Photon spectra appears thermal because the quarks and gluons creating the photons are in thermal equilibrium.

2. This is clear in a leading log approximation where

$$p^\mu_{\text{quark}} \approx q^\mu_{\text{photon}}$$

Photons from a viscous medium

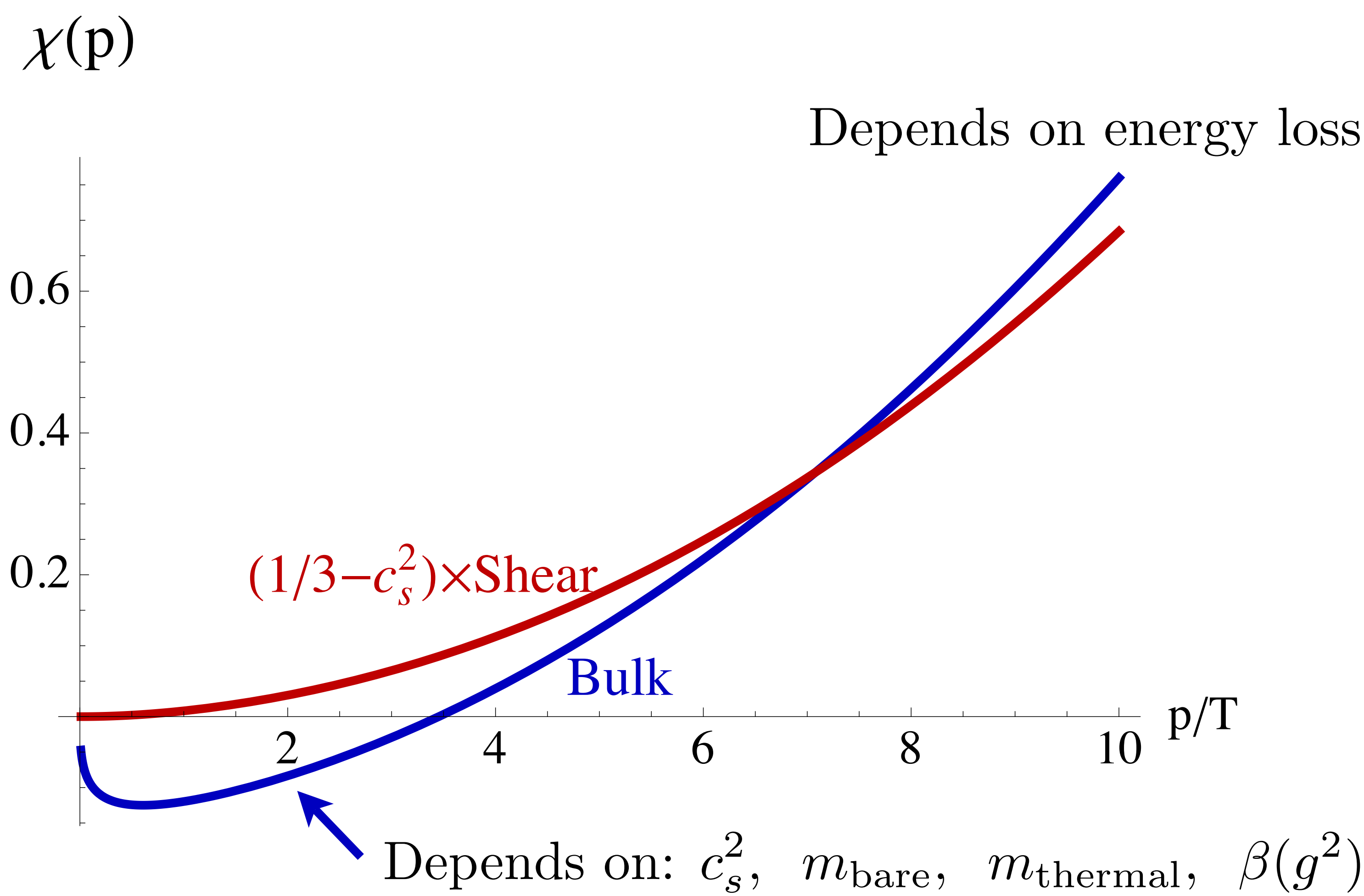
1. This simple kinetic theory calculations yields

$$E_\gamma \frac{dN_\gamma}{d^3q_\gamma} \propto \alpha_e \alpha_s \, f_{\rm quark}(q_\gamma)\, T^2 \ln\left(\# \frac{E_\gamma}{g^2 T}\right)$$

and depends on the quark's distribution function evaluated at the photon's momentum.

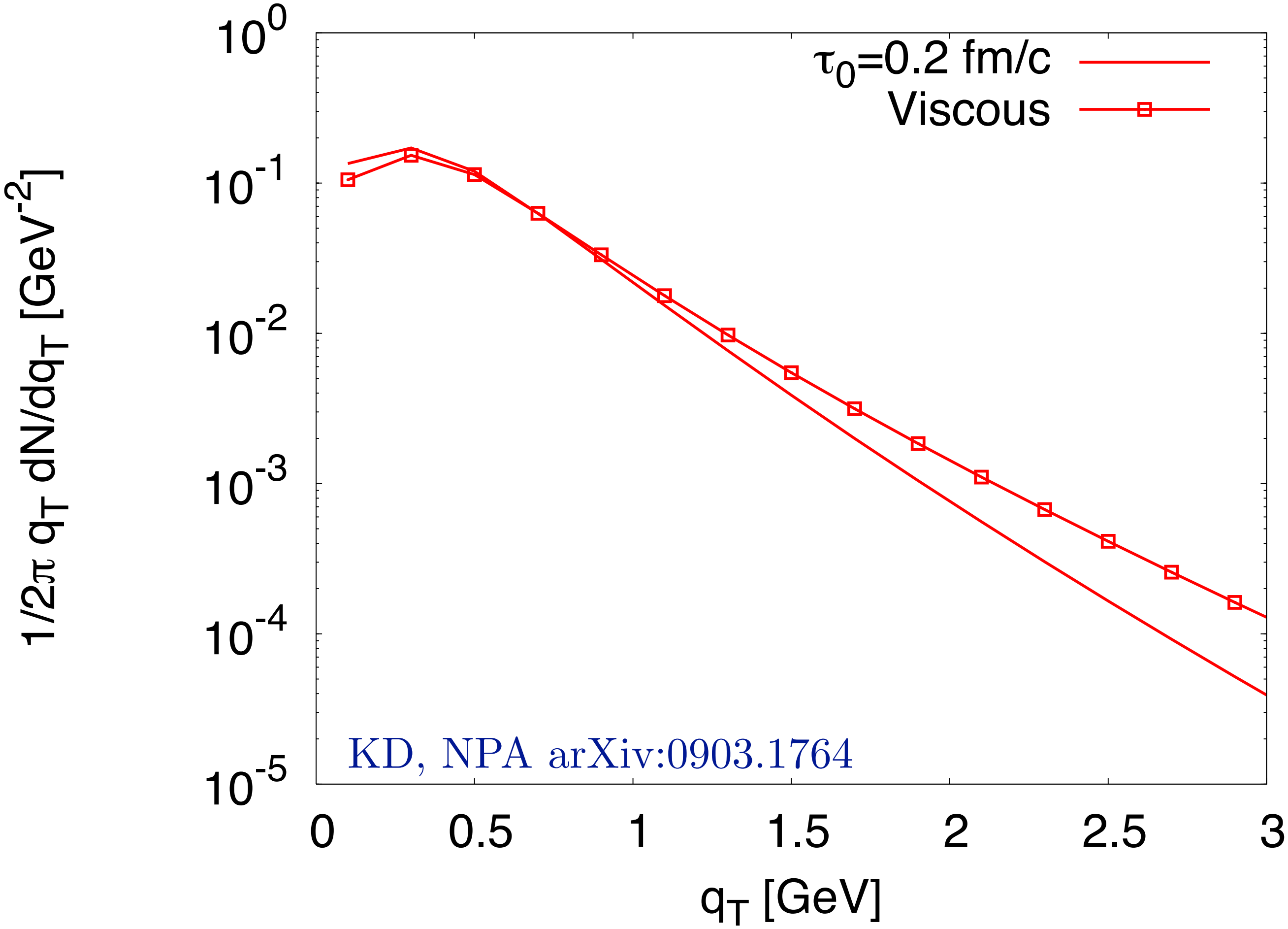
2. Out of equilibrium:

$$f_{\rm quark}(q_\gamma) = n_o(q_\gamma)\left(1 - \chi^{\pi}_{\rm quark}(q_\gamma) \cdot q^i_\gamma q^j_\gamma \partial_{\langle i} u_{j\rangle} - \chi^{\Pi}_{\rm quark}(q_\gamma) \cdot \partial_i u^i\right)$$



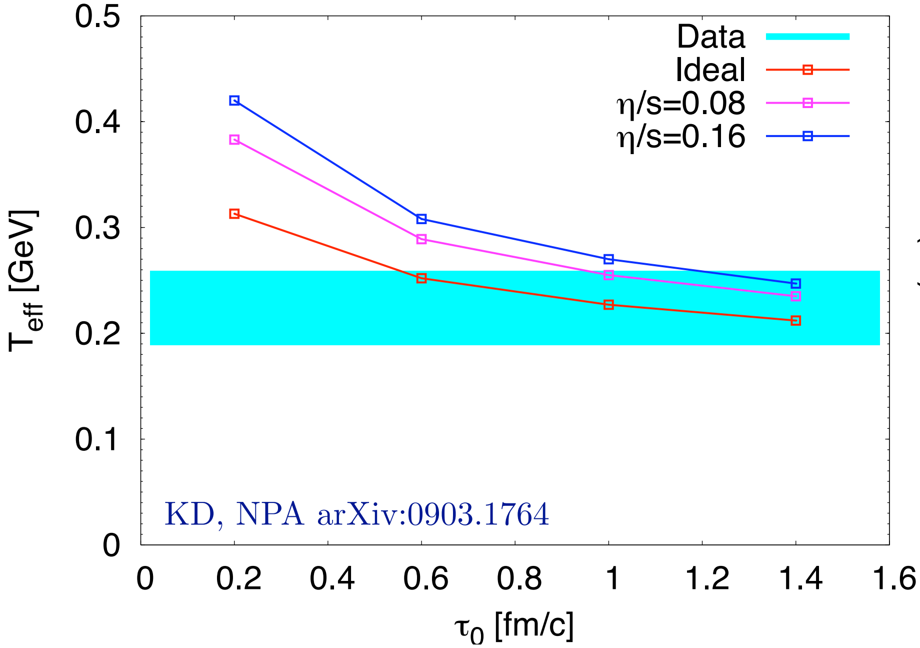
χ(p)
Depends on energy loss
0.6
0.4
0.2
$(1/3-c_s^2)\times$Shear
Bulk
p/T
2
4
6
8
10
Depends on: $c_s^2,\ m_{\rm bare},\ m_{\rm thermal},\ \beta(g^2)$

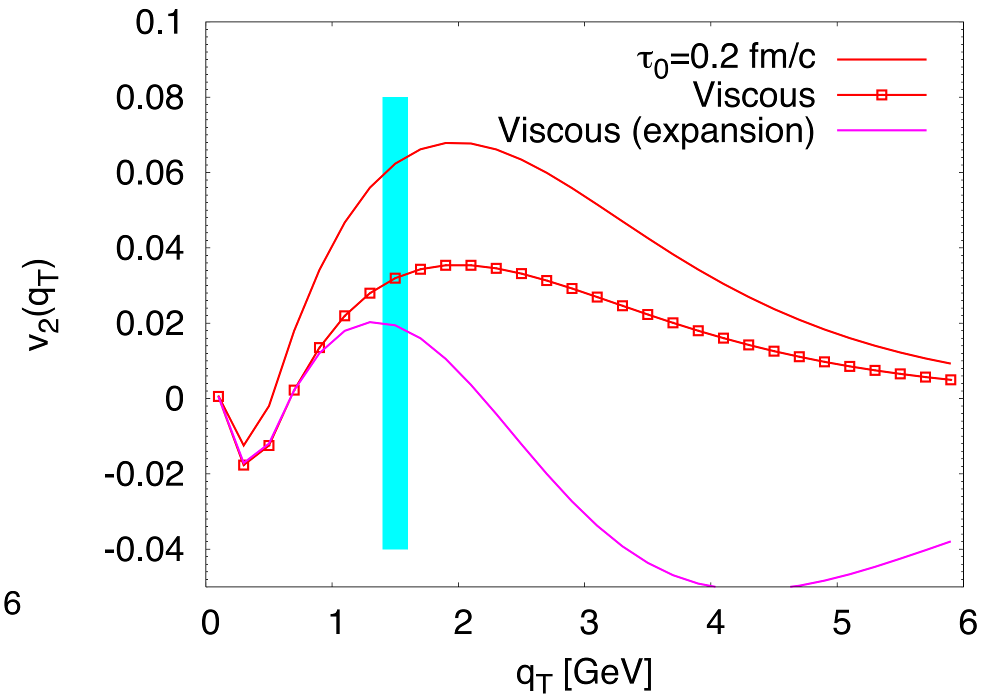
Photon Spectra (Shear viscosity only)



Viscosity increases Photon's effective temperature

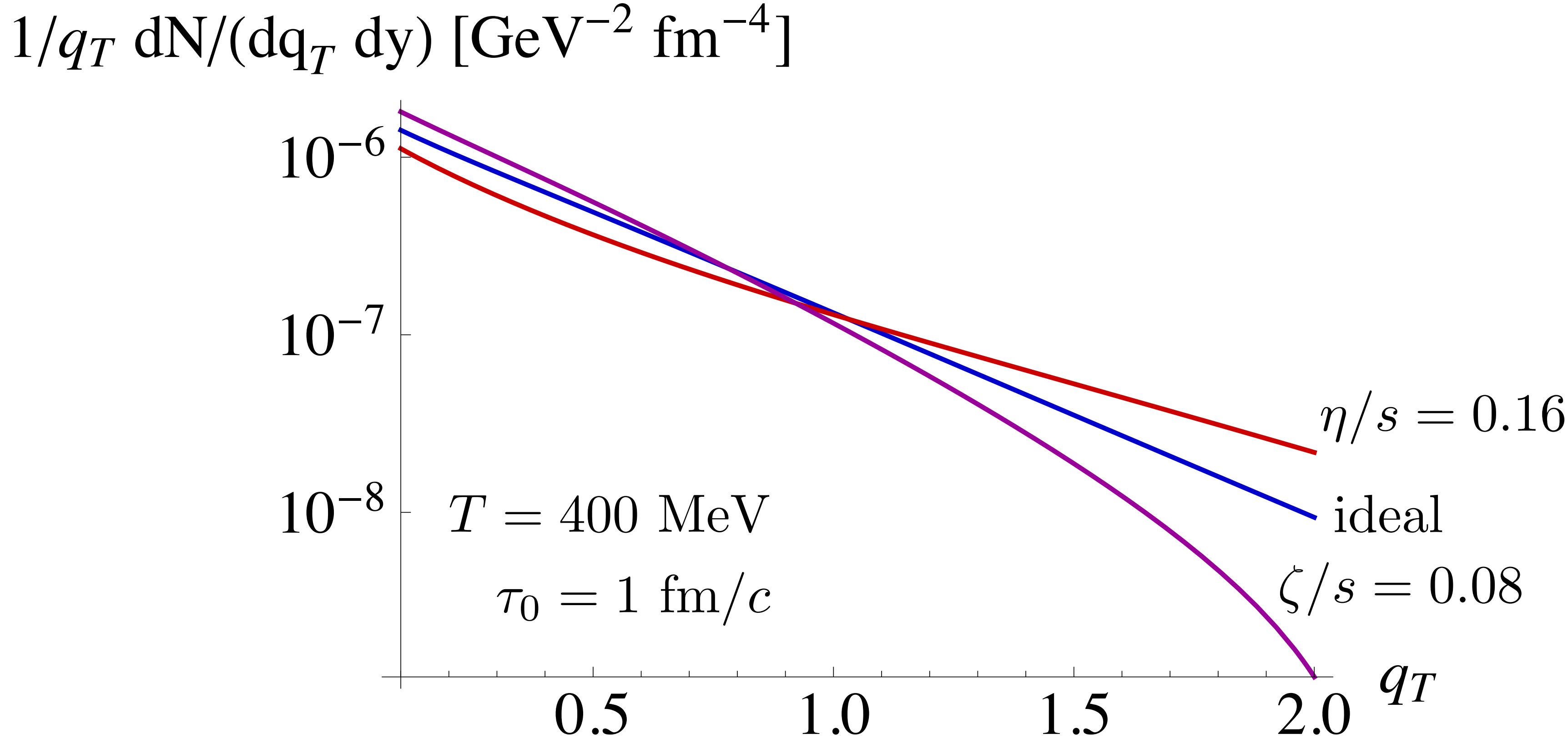
Photon "temperature" and elliptic flow (Shear viscosity only)



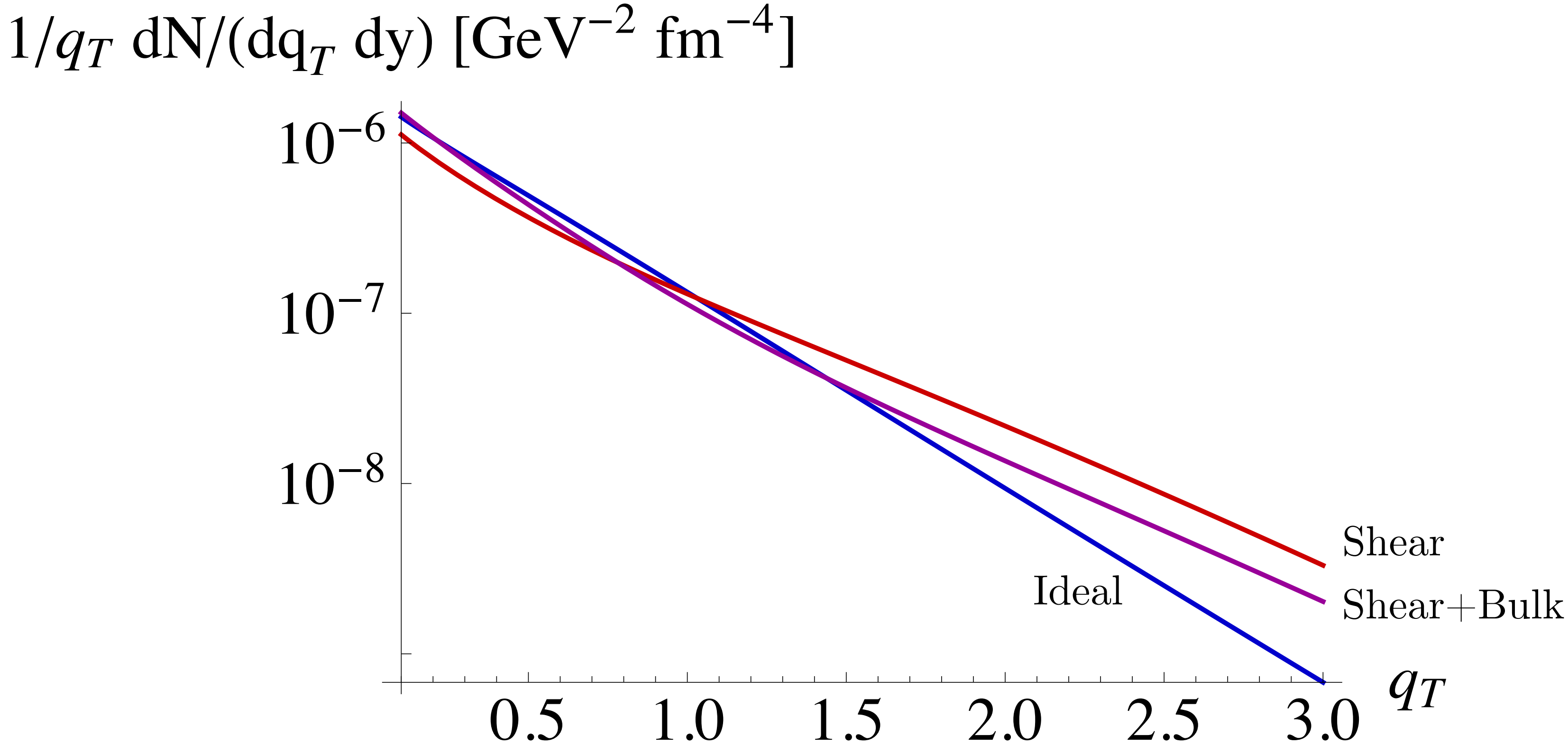


Lots of caveats here! Work in progress ...

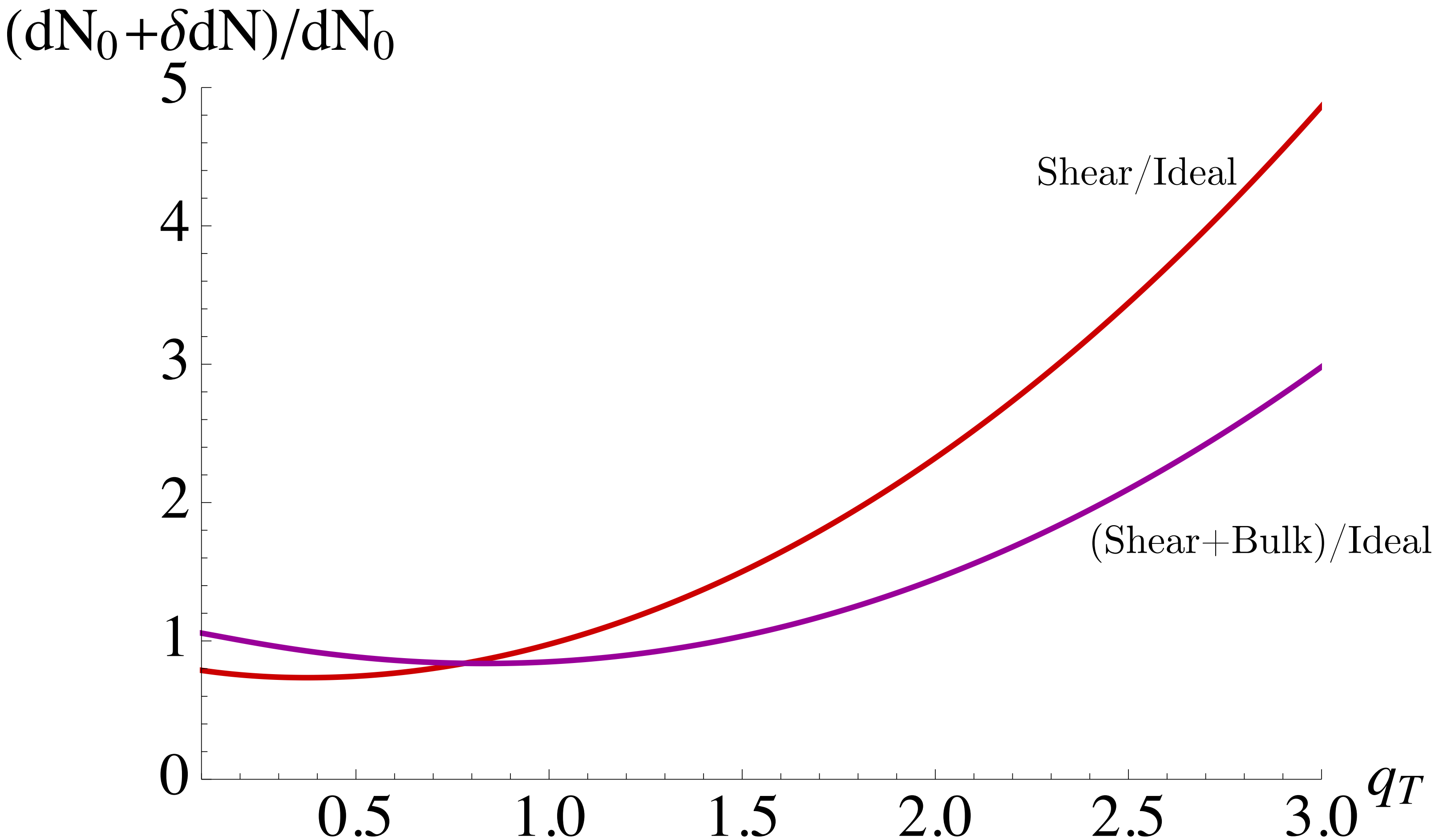
Now including bulk viscosity



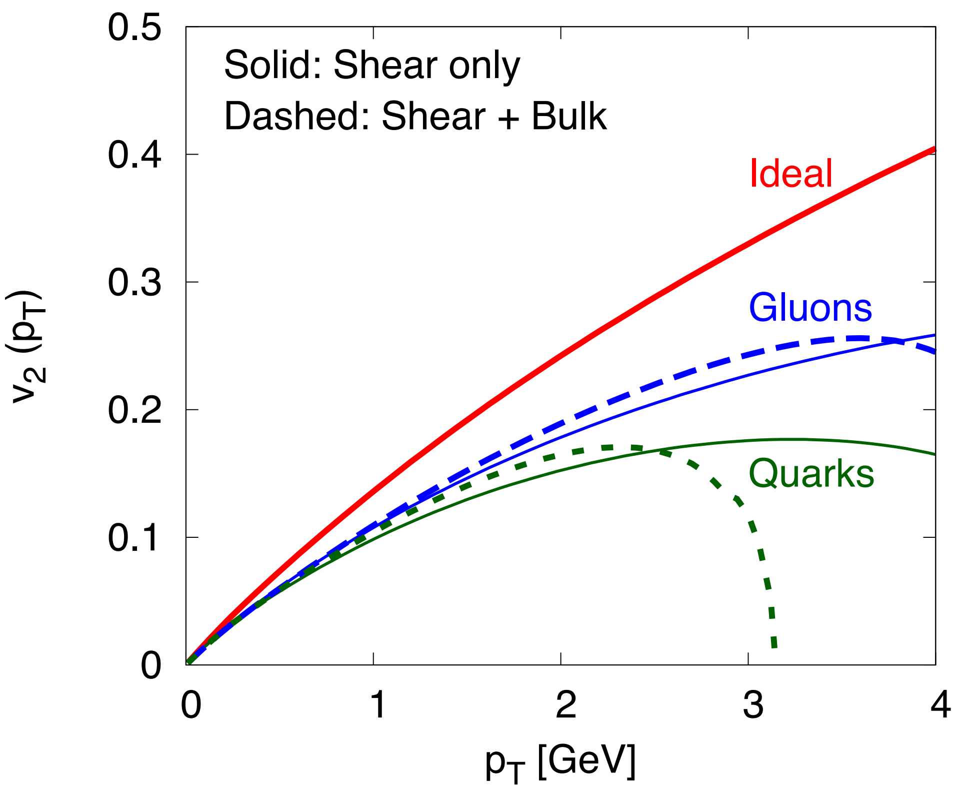
Now including bulk viscosity

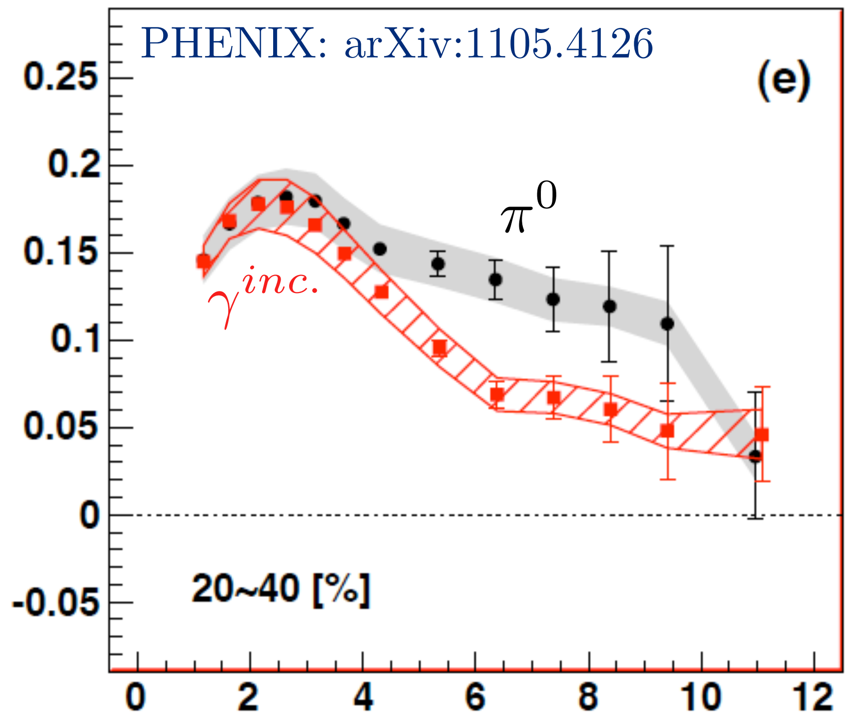


Now including bulk viscosity



Photon Elliptic Flow





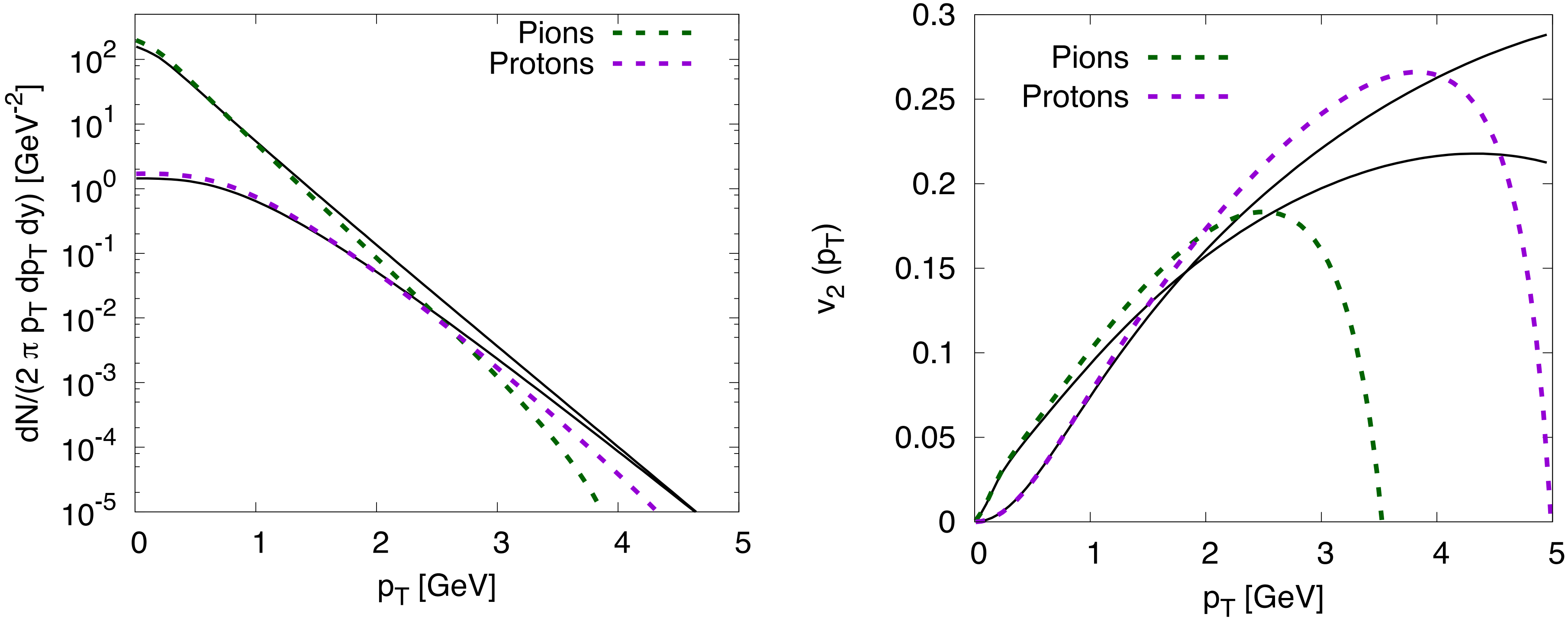
Can bulk viscosity explain the larger than expected photon elliptic flow?

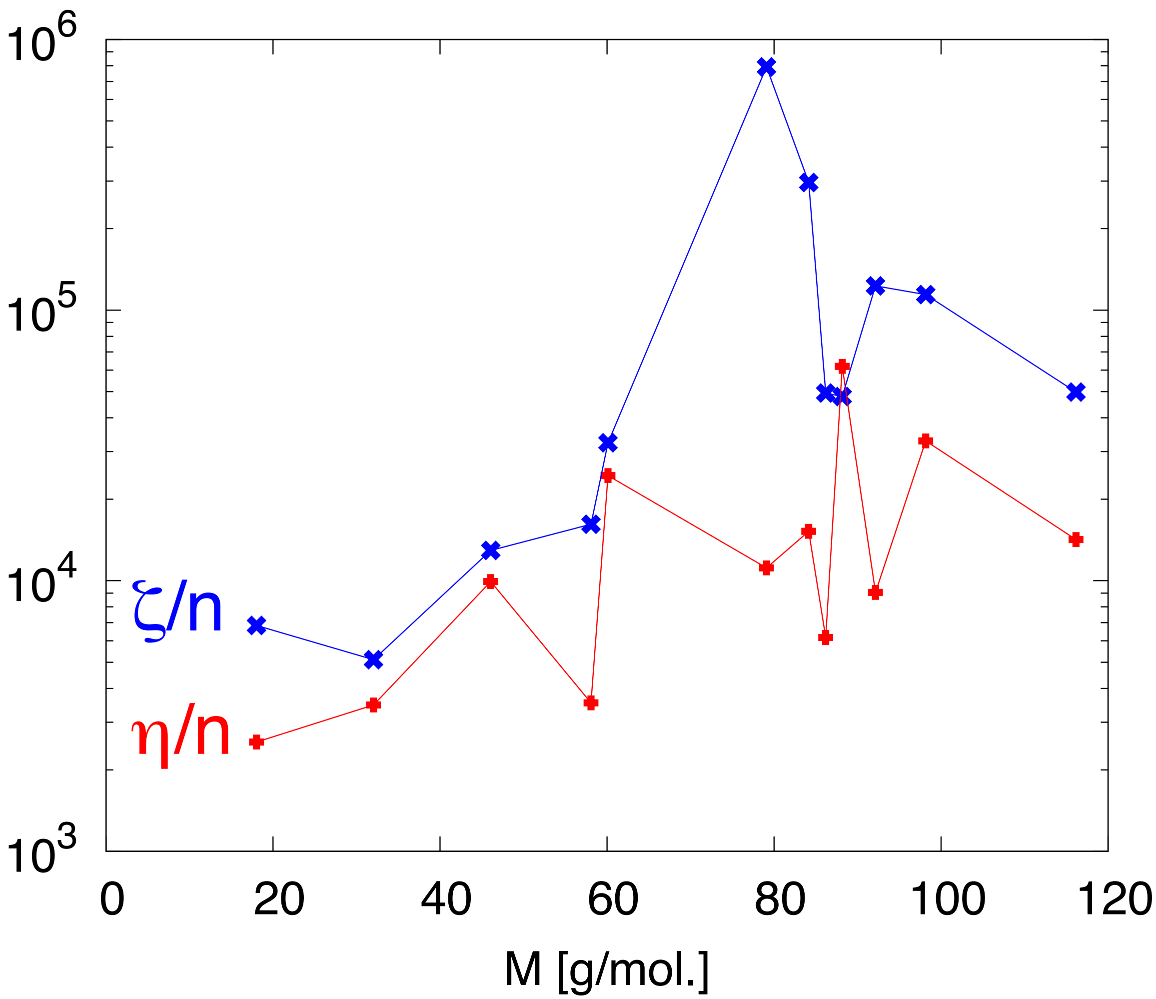
Conclusions

1. Data favors a non-vanished bulk viscosity and $\dfrac{\zeta}{s} \lesssim 0.05$
2. explain "pion wind" w/o resorting to RQMD
3. p_T integrated v_2 insensitive to bulk viscosity
4. There is promise for learning about transport from photons/di-leptons
5. Any constraints we can place on the bulk viscosity are important for cosmological models of the early universe

Backup

Pions / Protons





M [g/mol]	Liquid
18	Water
32	Methanol
46	Ethanol
58	Acetone
60	2-propanol
80	Pyridine
84	Cyclohexane
86	Hexane
88	1-pentanol
92	Toluene
98	Cyclohexanone
116	Butylacetate

Relaxation Time Approximation: Bulk

1. Again, start with the RTA

$$\mathcal{C}[f, \mathbf{p}] = \frac{f(\mathbf{p}) - n_{\mathbf{p}}}{\tau_R(E_{\mathbf{p}})}$$

2. The viscous correction is

$$\delta f = -\frac{\tau_R(E_{\mathbf{p}})}{E_{\mathbf{p}} T} n_{\mathbf{p}}(1 \pm n_{\mathbf{p}}) \partial_i u^i \left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}} \frac{\partial \left(\beta E_{\mathbf{p}}\right)}{\partial \beta} \right)$$

3. with the constraint

$$\Pi \equiv \frac{1}{3} T^{ii} - \mathcal{P}(\epsilon + \delta\epsilon) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 E_{\mathbf{p}}} \left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}} \frac{\partial \left(\beta E_{\mathbf{p}}\right)}{\partial \beta} \right) \delta f$$

Landau matching

1. But also the shift in energy density must vanish

$$\begin{aligned}\delta\epsilon_{\mathrm{RTA}} \quad \propto \quad & \int \frac{d^3\mathbf{p}}{(2\pi)^3} n_{\mathbf{p}}(1 \pm n_{\mathbf{p}}) \left(\frac{p^2}{3} - c_s^2 \tilde{E}_{\mathbf{p}}^2\right) (\beta E_{\mathbf{p}})^{1-\alpha} \\ & - \frac{\partial m^2}{\partial T^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} n_{\mathbf{p}}(1 \pm n_{\mathbf{p}}) \left(\frac{p^2}{3} - c_s^2 \tilde{E}_{\mathbf{p}}^2\right) (\beta E_{\mathbf{p}})^{-\alpha-1}\end{aligned}$$

2. which only happens in special circumstances

Re-summation

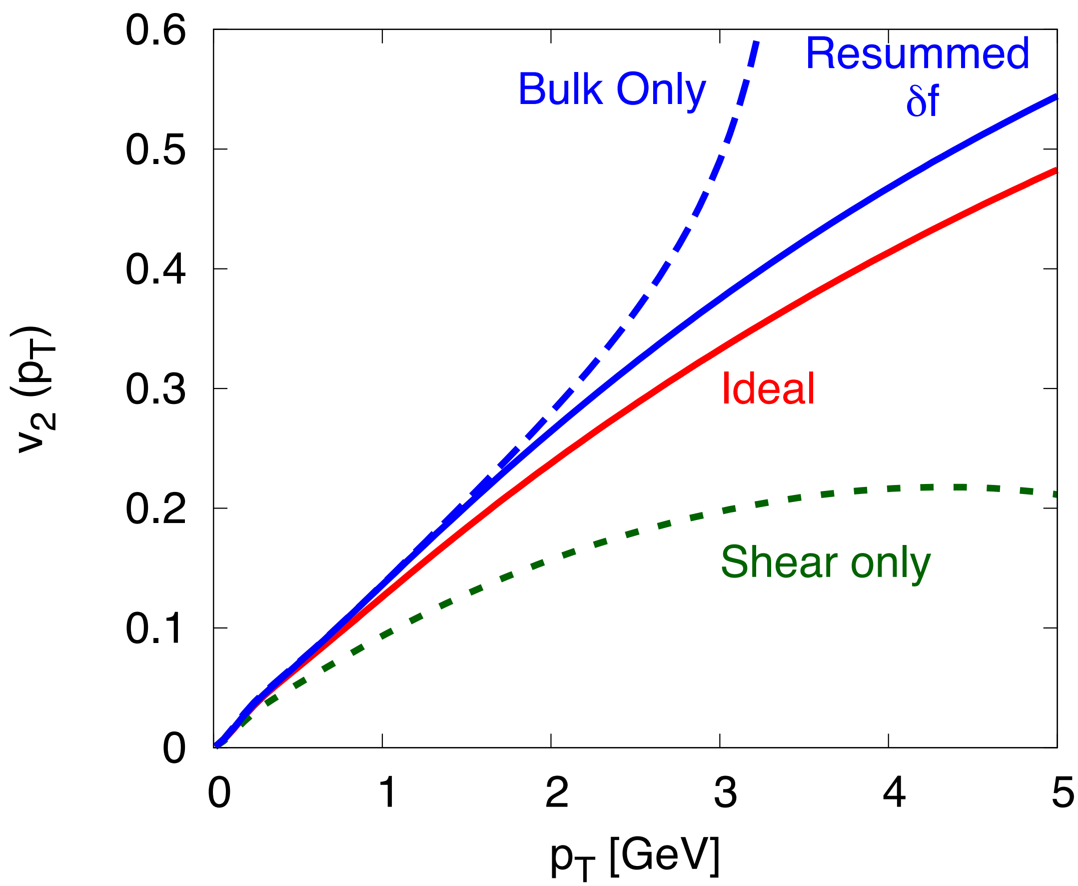
1. A taylor expansion of the equilibrium distribution shows the zero-mode coefficients are related to shifts in chemical potential and temperature

$$\delta f^a(\mathbf{p}) = n^a_{\mathbf{p}}(1 \pm n^a_{\mathbf{p}}) \left(\frac{\mu^a}{T} + \frac{E_{\mathbf{p}_a}\delta T}{T^2} \right) \qquad \begin{aligned} \mu^a &= -(\partial_k u^k) T \chi_0^a \\ \delta T &= +(\partial_k u^k) T^2 \chi_1 \end{aligned}$$

2. And we can make an ansatz for a re-summed δf

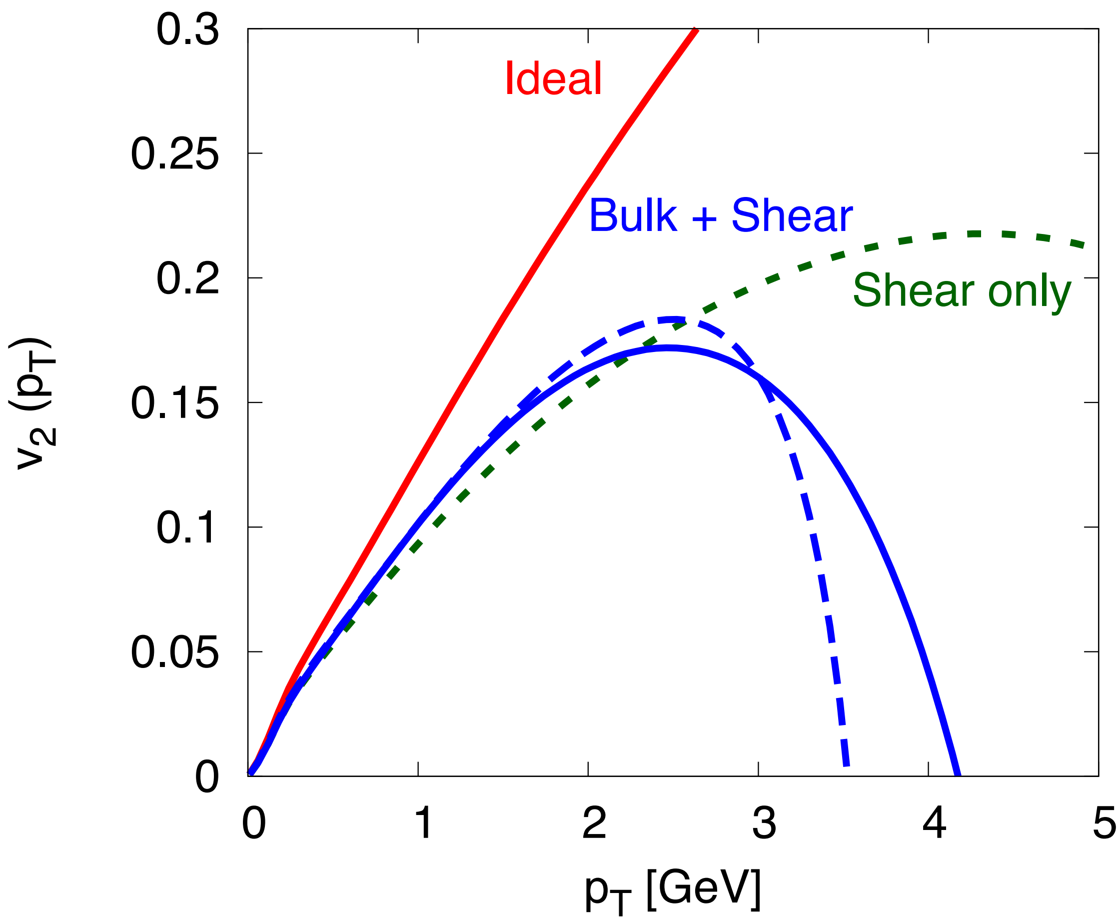
$$f^a(\mathbf{p}) \approx \frac{1}{e^{\frac{E_{\mathbf{p}_a}}{T+\delta T} - \beta\mu^a} \pm 1}$$

Pion: $v_2(p_T)$



0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
v_2 (p_T)
p_T [GeV]
Bulk Only
Resummed
δf
Ideal
Shear only

Pion: $v_2(p_T)$



Ideal
Bulk + Shear
Shear only
0.3
0.25
0.2
0.15
0.1
0.05
0
v_2 (p_T)
0
1
2
3
4
5
p_T [GeV]

Bulk channel at leading log

1. The Boltzmann equation can be recast as a Fokker-Plank equation at leading log order

$$\frac{1}{2}p^i p^j \sigma_{ij} + \partial_i u^i \left(\frac{p^2}{3} - c_s^2 E_{\mathbf{p}} \frac{\partial\,(\beta E_{\mathbf{p}})}{\partial \beta} \right) \quad = \quad \frac{T\mu_A}{n_{\mathbf{p}}(1+n_{\mathbf{p}})} \frac{\partial}{\partial \mathbf{p}^i} \left(n_{\mathbf{p}}(1+n_{\mathbf{p}}) \frac{\partial}{\partial \mathbf{p}^i} \left[\frac{\delta f(\mathbf{p})}{n_{\mathbf{p}}(1+n_{\mathbf{p}})} \right] \right)$$

2. This ode can be solved with appropriate boundary conditions

$$\frac{d\mathbf{p}}{dt} = \mu_A \hat{\mathbf{p}}\ , \qquad \mu_A = \frac{g^2 C_A m_D^2}{8\pi} \log\left(\frac{T}{m_D} \right)$$

Hong, Teaney PRC 82:044908, 2010.
and AMY, JHEP 0011:001 2000.

Transition Rates

$$
\begin{aligned}
C_{2\leftrightarrow 2}[\delta f, \mathbf{p}] &= \frac{1}{2!}\int_{\mathbf{k},\mathbf{p}',\mathbf{k}'} \Gamma_{\mathbf{pk}\to\mathbf{p}'\mathbf{k}'}\;\; n_{\mathbf{p}} n_{\mathbf{k}} (1+n_{\mathbf{p}'})(1+n_{\mathbf{k}'}) \\
&\qquad \times \left[\chi(p) + \chi(k) - \chi(p') - \chi(k')\right]\;, \\
C_{2\leftrightarrow 4}[\delta f, \mathbf{p}] &= \frac{1}{3!2!}\int_{\mathbf{k},\mathbf{p}',\mathbf{k}',\mathbf{q},\mathbf{q}'} \Gamma_{\mathbf{p}'\mathbf{k}\to\mathbf{pk}'\mathbf{qq}'}\;\; n_{\mathbf{p}} n_{\mathbf{k}'} n_{\mathbf{q}} n_{\mathbf{q}'} (1+n_{\mathbf{p}'})(1+n_{\mathbf{k}}) \\
&\qquad \times \left[\chi(p) + \chi(k) - \chi(p') - \chi(k') - \chi(q) - \chi(q')\right] \\
&- \frac{1}{4!1!}\int_{\mathbf{k},\mathbf{p}',\mathbf{k}',\mathbf{q},\mathbf{q}'} \Gamma_{\mathbf{pk}\to\mathbf{p}'\mathbf{k}'\mathbf{qq}'}\;\; n_{\mathbf{p}'} n_{\mathbf{k}'} n_{\mathbf{q}} n_{\mathbf{q}'} (1+n_{\mathbf{p}})(1+n_{\mathbf{k}}) \\
&\qquad \times \left[\chi(p') + \chi(k) - \chi(p) - \chi(k') - \chi(q) - \chi(q')\right]
\end{aligned}
$$

1. The $2 \leftrightarrow 2$ collision operator has an exact zero mode

$$\chi(p) = \chi_0 - \chi_1 E_p$$